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AN EXTENSION OF THE MOORA METHOD FOR SOLVING FUZZY DECISION MAKING PROBLEMS

Dragisa STANUJKIC

Faculty of Management Zajecar, Megatrend University Belgrade, Park Suma "Kraljevica" bb, 19000 Zajecar, Serbia

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Abstract. The aim of this paper is to propose an extension of the MOORA method to use the triangular fuzzy numbers. In this paper, several methods for defuzzification and calculation of the distance between two fuzzy numbers, are discussed. By applying these methods, the Ratio system and the Reference point approach of the MOORA method can be used in fuzzy environment. Thanks to the proposed modification, the MOORA method can be used to solve a greater number of real world problems. To demonstrate the applicability and effectiveness of the proposed approach, an example of grinding circuits design selection is considered.

Keywords: MCDM, MOORA, ratio system, reference point, uncertainty, triangular fuzzy numbers.

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Introduction

Multiple criteria decision making (MCDM) provides an opportunity for selecting the most acceptable alternative, based on conditions that are stated using the criteria. MCDM is a very popular and commonly used approach for selecting the most acceptable alternative from a wide range of available alternatives.

This approach has been used to solve various problems in many fields, which have been published in numerous professional and scientific journals. Some of them are supplier selection (Chen *et al.* 2006), new product launch strategy evaluation (Chiu *et al.* 2006), training

Corresponding author Dragisa Stanujkic
E-mail: *dragisa.stanujkic@fmz.edu.rs*

aircraft evaluation (Wang, Chang 2007), plant layout selection (Yang, Hung 2007), sustainable development strategy evaluation (Saparauskas 2007; Zavadskas *et al.* 2007), banking performances evaluation (Wu *et al*. 2009), solving many decision making problems in construction (Kaklauskas *et al*. 2006; Podvezko *et al*. 2010; Zavadskas *et al*. 2010a), and so on.

A number of methods have been proposed in the field of MCDM, such as Compromise programming (Zeleny 1973; Yu 1973), AHP (Saaty 1980), TOPSIS (Hwang, Yoon 1981), PROMETHEE (Brans, Vickine 1985), ELECTRE (Roy 1991), COPRAS (Zavadskas *et al*. 1994), VIKOR (Opricovic 1998) and ARAS (Zavadskas, Turskis 2010).

The Multi-Objective Optimization, on the basis of Ratio Analysis (MOORA) method, is a newly proposed method introduced by Brauers and Zavadskas (2006). Although the MOORA is a newly proposed method, it is applied to solve many economic, managerial and construction problems. For example, Brauers and Zavadskas (2010a, 2008) and Brauers and Ginevicius (2010, 2009) use the MOORA method for solving decision making problems in various fields of economy. Kracka *et al.* (2010) applies the MOORA method in construction in order to solve problems related to energy loss in heating buildings while, Chakraborty (2011) uses the MOORA method to solve different decision making problems in the real-time manufacturing environment. Based on Ratio system approach of the MOORA method, Gadakh (2011) proposed an optimization of the milling process, Karande and Chakraborty (2012) proposed the selection of the ERP system, and Dey *et al.* (2012) proposed both supplier selection and warehouse location selection.

Brauers and Zavadskas (2010a) also presented the MULTIMOORA method, as an extension of the MOORA method with a full multiplicative form. Like the MOORA method, MULTIMOORA method is also widely used for solving numerous problems, such as regional development (Brauers, Zavadskas 2010b, 2011a; Brauers, Ginevicius 2010), choice of bank loan (Brauers, Zavadskas 2011b) personnel selection (Balezentis *et al.* 2012a, b), and forming a multi-criteria decision making framework for prioritization of energy crops (Balezentiene *et al*. 2013).

The MOORA method, as well as many other ordinary MCDM methods, is based on the use of crisp numbers. Unfortunately, many real-world problems cannot be adequately represented using crisp numbers.

Many complex real-world problems, such as problems associated with uncertainty and problems which include some types of prediction, can be more adequately expressed by using grey or fuzzy numbers. Therefore, a lot of ordinary MCDM methods are also extended for using the interval grey of fuzzy numbers. Some proposed extensions of ordinary MCDM methods are: Grey TOPSIS (Zavadskas *et al.* 2010b; Chen, Tzeng 2004), COPRAS-G (Zavadskas *et al*. 2008, 2009), COPRAS-F (Zavadskas, Antucheviciene 2007), ARAS-G (Turskis, Zavadskas 2010a), ARAS-F (Turskis, Zavadskas 2010b), SAW-G (Zavadskas *et al*. 2010b; Medineckiene *et al*. 2010) and many extensions of the Fuzzy TOPSIS method (Wang, Elhag 2006; Wang, Chang 2007; Saremi *et al.* 2009).

Similar to other MCDM methods, for the MOORA and the MULTIMOORA have proposed some extensions. Brauers *et al.* (2011) proposed first fuzzy extension of the MOORA method, or more precisely MULTIMOORA method. In this extension the MULTIMOORA method was updated with the fuzzy number theory, and all three parts of the MULTIMOORA

method: Ratio system, Reference point and Full multiplicative form were modified to enable the usage of triangular fuzzy numbers. Balezentis *et al.* (2012a, b) further modified the fuzzy MULTIMOORA, and proposed the fuzzy extension, named MULTIMOORA-FG, which include the use of linguistic variables and the group decision making. Balezentis and Zeng (2013) also proposed an extension of MULTIMOORA based on interval-valued fuzzy numbers.

Karande and Chakraborty (2012), and Dey *et al.* (2012) proposed the fuzzy extensions of Ratio system approach of the MOORA method. Both of these extensions enabled the use of fuzzy triangular numbers. The extension proposed by Dey *et al.* (2012) also included the group decision making, but in this approach decision matrix was defuzzified at the initial stage, and then the crisp MOORA was further employed.

Stanujkic *et al.* (2012a, b) considered a grey extension of the MOORA method. Stanujkic *et al.* (2012b) proposed a simple to use grey extension of the Ratio system part of the MOORA method. In Stanujkic *et al.* (2012a) the grey extensions of both approaches of the MOORA method, Ratio system and Reference point approaches, were discussed in details. In this paper significant attention was given to the transformation of grey into the crisp numbers.

Brauers and Zavadskas (2012) provided a comprehensive comparison of prominent MCDM methods. Based on this comparison, and the use of MOORA method that are published in journals, it can be concluded that the MOORA method is very efficient and relatively easy to use. Therefore, in this paper an extension of MOORA methods is proposed in order to allow the use of triangular fuzzy numbers, and thus ensure its application to solve a great number of decision making problems.

Stanujkic *et al.* (2012b) state that more benefits from the use of fuzzy numbers can be achieved if they are transformed into crisp numbers in the later stages of the MCDM process. Therefore, in this paper the particular attention is given to the use of different methods for defuzzification as well as calculation of the distance between fuzzy numbers, in the later stages of the MCDM process. Although there are significant similarities with Brauers *et al.* (2011) and Balezentis *et al.* (2012a, b), this paper differs in terms of normalization and distance measurement techniques.

Because of all above mentioned reasons, the rest of this manuscript is organized as follows. In section 1, the basic elements of the fuzzy system theory are considered. In Section 2, the ordinary MOORA method is presented, and in Section 3 the fuzzy extension of MOORA method is presented. In Section 4, a case study is considered with the aim to explain in details the proposed methodology. Finally, the conclusions are given.

1. Fuzzy set theory

The classical MCDM methods are based on the use of the classical set theory, where an element can belong or does not belong to the set. Let A be a classical set of objects, called the universe, whose generic elements are denoted by x. The belonging to a set A can be represented by membership functions μ_A, which has the following form (Jahanshahloo *et al.* 2006):

$$\mu_A(x) = \begin{cases} 1 & x \in A, \\ 0 & x \notin A. \end{cases} \tag{1}$$

Unfortunately, many real-world decision making problems are often related to the impact of uncertainty, which cannot be easily expressed using the classical sets.

Zadeh (1965) introduced the Fuzzy sets theory, which allows a partial membership in a set. As a result, instead of the exclusive use of crisp numbers, the fuzzy set theory allows the use of other forms of numbers, such as triangular, trapezoidal, and bell-shaped numbers. In addition, an approach for the formalization of natural language specification, called computation with words, was established as an extension of the fuzzy set theory.

1.1. The triangular fuzzy numbers

A triangular fuzzy number (TFN), shown in Figure 1, is fully characterized by a triplet of real numbers (l,m,u), where parameters *l*, *m*, and *u*, indicate the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event (Dubois, Prade 1980; Ertugrul, Karakasoglu 2009). The most promising value of TFN is often called mode or core.

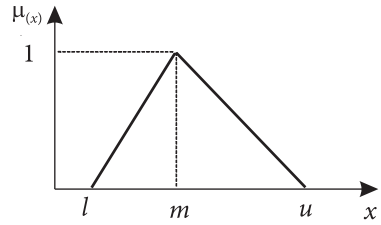

Fig. 1. Triangular fuzzy number

The membership function of the TFN is defined as:

$$\mu(x)=\begin{cases}0 & x<l\\ (x-l)/(m-l) & l\le x\le m\\ (u-x)/(u-m) & l\le x\le u\\ 0, & x>u\end{cases}. \tag{2}$$

As important characteristics of a TFN there can also be specified: mode *m*, support $(u-l)$, left spread $(m-l)$ and right spread $(u-m)$. TFN with equal left and right spread is known as a symmetrical TFN (STFN).

Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers, parameterized by a triple (a_l, a_m, a_u) and (b_l, b_m, b_u) respectively. Then, the basic operations on these fuzzy numbers are defined as (Dubois, Prade 1980; Wang, Chang 2007):

$$\tilde{A}+\tilde{B}=(a_l+b_l,\ a_m+b_m,\ a_u+b_u); \tag{3}$$

$$\tilde{A}-\tilde{B}=(a_l-b_u,\ a_m-b_m,\ a_u-b_l); \tag{4}$$

$$\tilde{A}\times\tilde{B}=(a_l b_l,\ a_m b_m,\ a_u b_u); \tag{5}$$

$$\tilde{A}\div\tilde{B}=\left(\frac{a_l}{b_u},\frac{a_m}{b_m},\frac{a_u}{b_l}\right). \tag{6}$$

The following unary operations on triangular fuzzy numbers are also important:

$$k \times \tilde{A} = (ka_l,\ ka_m,\ ka_u); \tag{7}$$

$$\tilde{A}^{-1} = \left(\frac{1}{a_u}, \frac{1}{a_m}, \frac{1}{a_l} \right). \tag{8}$$

1.2. Linguistic variable

In a series of papers, Zadeh (1975a, b, c) introduced the concept of linguistic variables. According to Zadeh, the linguistic variables are defined as variables whose values are words or sentences in a natural or artificial language.

The concept of linguistic variable is very suitable for dealing with many real-world problems, which are usually complex, slightly defined and related with uncertainties. The exclusive use of crisp numbers to represent responses of alternatives on objectives (also known as performance ratings of alternatives, in some other MCDM methods) and/or significance coefficients (also known as weights of criteria or criteria weights, in other MCDM methods) when solving complex real-world problems requires some kind of averaging. In contrast, the use of linguistic variables, which are represented with corresponding fuzzy numbers (Fig. 2), in such case is more appropriate.

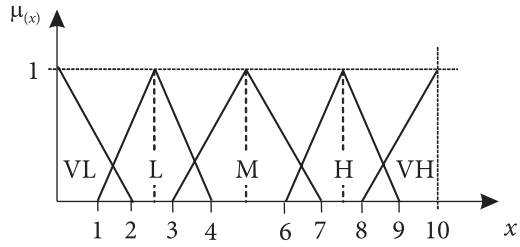

Fig. 2. The membership functions of linguistic variables

In literature, numerous studies have considered the use of numerous linguistic scales. Unlike many other approaches, in this paper it has been proposed the use of same scale for assigning significance coefficients and responses of alternatives. The proposed linguistic scale is shown in Table 1.

Table 1. Linguistic scales for significance coefficients and responses of alternatives

Linguistic variable	Corresponding TFN	TFN support
Very low (VL)	(0.00, 0.00, 2.00)	2
Low (L)	(1.00, 2.25, 4.00)	3
Medium (M)	(3.00, 5.00, 7.00)	4
High (H)	(6.00, 7.00, 9.00)	3
Very high (VH)	(8.00, 1.00, 1.00)	2

In this approach, fuzzy numbers have different supports and therefore they provide a higher significance to the moderate attitudes. About the proposed approach it can be discussed, but it is estimated that it provides a positive impact on decision makers behaviour, during evaluation of significance coefficients and responses of alternatives, because the use of different supports stimulate decision makers to perform evaluation more carefully.

1.3. Defuzzification

As a result of performing an operation on fuzzy numbers, the obtained result is also a fuzzy number. Therefore, in order to rank alternatives in fuzzy environment using MCDM methods, these methods must be able to perform the ranking based on overall fuzzy responses, or must transform overall fuzzy responses into crisp responses before they perform ranking.

Over time, a number of different methods for ranking fuzzy numbers and/or defuzzification are proposed. The first method for ranking fuzzy numbers was proposed by Jain (1976). Since then, a number of methods, with different complexity, have been proposed.

From these methods, in this subsection, a few simple, understandable and easy to use defuzzification methods have been considered.

For mapping a fuzzy into a corresponding crisp number, Kaufmann and Gupta (1988) proposed Eq. (9):

$$gm(\tilde{A}) = (l + 2m + u)/4, \tag{9}$$

with $gm(\tilde{A})$ as a resulting crisp number, i.e. the generalized mean of fuzzy number $\tilde{A}$.

In Kaufman and Gupta's approach, if two fuzzy numbers have the same value of resulting crisp number then fuzzy number with the larger mode will be ranked higher. Also, if they have the same mode, the higher-ranked fuzzy number will be the one which has a smaller left spread.

Liou and Wang (1992) proposed the Interval Value method for ranking fuzzy numbers, and for calculating the generalized mean of fuzzy number they suggested Eq. (10):

$$gm(\tilde{A}) = \frac{1}{2}\left[(1-\lambda)\ l + m + \lambda\ u\right], \tag{10}$$

with λ as a coefficient which represents the decision maker risk-taking attitude, also denoted as an index of optimism, and $\lambda \in [0,1]$.

For mapping a fuzzy into a corresponding crisp numbers Chiu and Park (1994) proposed Eq. (11):

$$gm(\tilde{A}) = \frac{1}{3}(l + m + u) + \lambda m, \tag{11}$$

with λ as a coefficient by which the decision maker can express his opinion about the nature and importance of the TFN mode, and $\lambda \geq 0$.

And finally, to determine the generalized mean of fuzzy number, Opricovic and Tzeng (2003) proposed Eq. (12):

$$gm(\tilde{A}) = l + \frac{(m-l) + (u-l)}{3}, \tag{12}$$

which can be also expressed by Eq. (13):

$$gm(\tilde{A}) = \frac{l+m+u}{3} \ . \tag{13}$$

1.4. The distance between fuzzy numbers

In order to rank alternatives, in some cases, it is necessary to determine the distance between two fuzzy numbers. One of the probably most prominent methods for determining the distance between fuzzy numbers is the Vertex method (Chen 2000).

Let $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ be two triangular fuzzy numbers, then the Vertex method is defined to calculate the distance between them, as follows:

$$d_{vert}(\tilde{A}, \tilde{B}) = \left(\frac{1}{3}[(a_l - b_l)^2 + (a_m - b_m)^2 + (a_u - b_u)^2] \right)^{\frac{1}{2}} . \tag{14}$$

In addition to the known Vertex method in this paper the use of the maximum distance between fuzzy numbers is also proposed. Let $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ be two triangular fuzzy numbers, then the maximum distance is defined to calculate the distance between them, as follows:

$$d_{\max}(\tilde{A}, \tilde{B}) = \max\left(\left| a_1 - b_l \right|, \left| a_m - b_m \right|, \left| a_u - b_u \right| \right). \tag{15}$$

2. The MOORA method

The MOORA method is introduced by Brauers and Zavadskas (2006) on the basis of previous researches (Brauers 2004a, b).

In comparison to other MCDM methods, the MOORA method is specific, because it consists of two components: the Ratio system and the Reference point approach.

2.1. The ratio system approach of the MOORA method

In the Ratio system approach, the normalized responses are added in the case of maximization, i.e. benefit criteria[1], and subtracted in the case of minimization, i.e. cost criteria[2]. In other words, the overall performance of each alternative is calculated as a difference between the sum of normalized responses which belongs to benefit and the sum of normalized responses which belongs to cost criteria, which can be expressed by Eq. (16):

$$y_j^* = \sum_{i=1}^{g} x_{ij}^* - \sum_{i=g+1}^{i=n} x_{ij}^* \ , \tag{16}$$

where: x_{ij}^* as a normalized response of alternative j on objective i; $i = 1, 2, ..., g$ as the objectives to be maximized; $i = g + 1, g + 2, ..., n$ as the objectives to be minimized; $j = 1, 2, ..., m$ as the alternatives; and y_j^* as the overall ranking index of alternative j.

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[1] criteria to be maximized, i.e. the larger the better type.

[2] criteria to be minimized, i.e. the smaller the better type.

For further simpler presentations, the Eq. (16) is proposed:

$$y_j^* = y_j^+ - y_j^- \ , \tag{17}$$

where:

$$y_j^+ = \sum_{i \in \Omega_{\max}} x_{ij}^* \ ; \text{ and} \tag{18}$$

$$y_j^- = \sum_{i \in \Omega_{\min}} x_{ij}^* \ , \tag{19}$$

where: y_j^+ as a sum of normalized response of alternative *j* on objectives to be minimized; y_j^- as a sum of normalized response of alternative *j* on objectives to be minimized; $\Omega_{\max}$ as a set of objectives to be minimized; and $\Omega_{\min}$ as a set of objectives to be minimized.

Based on the Ratio system approach of the MOORA method, the optimal alternative A_{RS}^* can be determined using Eq. (20):

$$A_{RS}^* = \left\{ A_j \left| \max_j y_j^* \right. \right\}. \tag{20}$$

2.2. The reference point approach of the MOORA method

After a brief review of the most prominent reference point approaches, Brauers and Zavadskas (2006, 2009), Brauers *et al.* (2008) and Brauers (2008) emphasize that the Tchebycheff Min-Max metric (Karlin, Studden 1966) is the most appropriate.

Therefore, for optimization based on the Reference Point approach Brauers and Zavadskas (2006) proposed Eq. (21):

$$\min_j \left\{ \max_i \left| r_i - x_{ij}^* \right| \right\}, \tag{21}$$

where: r_i as *i*th coordinate of the reference point, i.e. the most desirable response of all alternatives with respect to objective *i*; x_{ij}^* as the normalized response of alternative *j* on objective *i*; $i = 1, 2, ..., n$ as the objectives; and $j = 1, 2, ..., m$ as the alternatives.

For further simpler presentations, it can be marked the distance from an alternative to the reference point with d and transform the Eq. (21) in the following form:

$$\min_j d_j^{\max}, \tag{22}$$

where:

$$d_j^{\max} = \max_i \left| r_i - x_{ij}^* \right| ; \text{ and} \tag{23}$$

$$r_i = \begin{cases} \max\limits_j x_{ij}^* ; \ i \in \Omega_{\max}, \\ \min\limits_j x_{ij}^* ; \ i \in \Omega_{\min}. \end{cases} \tag{24}$$

where: $d_j^{\max}$ as a maximum unsigned distance of alternative *j* to the reference point.

Based on equations (21) and (23), the optimal alternative in the Reference point approach of the MOORA method A_{RP}^* can be determined by Eq. (25):

$$A_{RP}^* = \left\{ A_j \left| \min_j d_j^{\max} \right. \right\}. \tag{25}$$

2.3. The normalization procedure

Equations (16) and (21) use normalized responses of alternatives. Brauers and Zavadskas (2006) proved that the most robust choice for the denominator is the square root of the sum of squares of each alternative per objective, and therefore the use of the vector normalization method is recommended in order to normalize responses of alternatives. As a result, the following equation proposed by Van Delft and Nijkamp (1977) was used:

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^{2}}}, \tag{26}$$

where: x_{ij}^{*} as a normalized response of alternative j on objective i; and $x_{ij}^{*} \in [0, 1]$.

2.4. The importance given to objectives

When solving the real-world problems using MCDM methods, objectives mainly do not have the same significance, i.e. some objectives are more important than the others. However, Eqs. (16) and (21) do not give a possibility to express a different significance of objectives.

Therefore, to include different significance of objectives (Brauers, Zavadskas 2009, 2012; Chakraborty 2011) introduced the Significance Coefficient.

To include the significance coefficient in the Ratio system approach of the MOORA method, Brauers and Zavadskas (2009) adapted the Eq. (16), and to calculate a overall ranking index of alternatives they proposed the Eq. (27):

$$\ddot{y}_{j}^{*} = \sum_{i=1}^{g} s_i x_{ij}^{*} - \sum_{i=g+1}^{i=n} s_i x_{ij}^{*}, \tag{27}$$

where: s_i as the significance coefficient of objective i; and $\ddot{y}_{j}^{*}$ as an overall ranking index of alternative j with respect to all objectives and their significance coefficients, $\ddot{y}_{j}^{*} \in [-1, 1]$.

A simple way to include the different significance given to objectives into the Ratio system approach of the MOORA method is to use the Eq. (17), and instead of the Eq. (18) and Eq. (19), use their adapted forms, as follows:

$$y_{j}^{+} = \sum_{i \in \Omega_{\max}} s_i x_{ij}^{*}; \text{ and} \tag{28}$$

$$y_{j}^{-} = \sum_{i \in \Omega_{\min}} s_i x_{ij}^{*}. \tag{29}$$

By using equations (28) and (29), instead of equations (18) and (19), when objectives have a different significance, the remaining part of the Ratio system of the MOORA, shown using the equations (17) and (20), remains the same.

To include the significance coefficient in the Reference Point approach of the MOORA method, Brauers and Zavadskas (2009) adapted the equation (21), and introduce it in the following form:

$$\min_{j} \left\{ \max_{i} \left| s_i r_i - s_i x_{ij}^{*} \right| \right\}. \tag{30}$$

A simple way to include the different significance given to objectives into Reference point approach of the MOORA method is to use the Eq. (22), and instead of the Eq. (23) use Eq. (31):

$$d_j^{\max} = \max_i \left\{ s_i \left| r_i - x_{ij}^* \right| \right\}. \tag{31}$$

After that, the Eq. (25) still remains without changes for determining the most appropriate alternative based on the Reference point approach of the MOORA method.

3. The proposed methodology

A systematic approach to extend the MOORA method to solve decision making problems in fuzzy environment is proposed in this section. The first part covers the steps that are common for both approaches. After that, the second and the third parts deal with the steps that are related to the Ratio system and Reference point approaches of the MOORA method.

3.1. The common steps of the Fuzzy MOORA method

As initial steps in solving decision making problems by using MCDM methods can be identified:

- Identify alternatives, which can be used to solve problem; and
- Select objectives, on which basis the evaluation of alternatives will be done.
- Next usually follow the typical steps, such as:
- Determine the responses of alternatives on objectives, and construct a decision matrix;
- Determine the significance of the objectives; and
- Normalize the responses of alternatives.

In relation to the steps in ordinary MCDM methods, the use of fuzzy numbers and linguistic variables has certain specificities, and these will be discussed below.

3.1.1. Determine the responses of alternatives on objectives, and construct a fuzzy decision matrix

A fuzzy MCDM selection problem, which involves *m* alternatives, *n* objectives and *K* decision makers, can be expressed in the following matrix form:

$$\tilde{D}^k = [\tilde{x}_{ij}^k]_{m \times n}, \tag{32}$$

where: $\tilde{D}^k$ as a fuzzy decision matrix formed by decision maker/expert *k*; $\tilde{x}_{ij}^k$ as a fuzzy response of alternative *j* on objective *i* given by decision maker *k*, using linguistic variables from Table 1; $i = 1, 2, ..., n$ as the objectives; $j = 1, 2, ..., m$ as the alternatives; and $k = 1,2, \ldots, K$ as the decision makers and/or experts.

As a result of the use of the Eq. (39) we have a *k* decision matrix. To form the resulting fuzzy decision matrix $\tilde{D}$:

$$\tilde{D} = [\tilde{x}_{ij}]_{m \times n}, \tag{33}$$

the following equation was used:

$$\tilde{x}_{ij} = \frac{1}{K}\sum_{k=1}^{K}\tilde{x}_{ij}^{k}\,, \tag{34}$$

where: $\tilde{x}_{ij}$ as a fuzzy response of alternative j on objective i.

While forming the resulting fuzzy decision matrix, the linguistic variables are also transformed into the corresponding triangular fuzzy numbers.

3.1.2. Determine the significance of objectives

In MCDM, the significance coefficient is very important, because it has a great impact on the selection of the most acceptable alternative. Consequently, in literature, there is proposed a number of different approaches for its determination, such as pairwise comparisons taken from the AHP method, the Entropy method and so on.

To determine the significance coefficient more realistic often is necessary to take into account the opinions of several experts. In such cases, the use of linguistic variables can be very appropriate.

For a decision making problem which involves n objectives and K decision makers, the fuzzy significance coefficient can be calculated using the Eq. (35):

$$\tilde{s}_i^{,} = \frac{1}{K}\sum_{k=1}^{k}\tilde{s}_i^{k}\,, \tag{35}$$

where: $\tilde{s}_i^{,}$ as the non-normalized fuzzy significance coefficient of objective i; $\tilde{s}_j^{k}$ is the fuzzy significance coefficient i given by decision maker k using data from Table 1; $i = 1, 2, ..., n$ as the objectives; and $k = 1, 2, \ldots, K$ as the decision makers and/or experts.

In ordinary MCDM methods the significance of objectives expressed by using significance coefficients satisfy the following condition:

$$\sum\nolimits_{i=1}^{n} s_i = 1\,. \tag{36}$$

Fuzzy significance coefficients obtained by using the Eq. (35) do not satisfy the condition (36), and therefore they must be scaled, i.e. normalized. If we denote $\tilde{s}_i^{,}$ as $(s_{im}^{,}, s_{im}^{,}, s_{iu}^{,})$ then the procedure proposed for scaling (normalizing) non-normalized significance coefficients obtained by using Eq. (35) can be represented as follows:

$$\tilde{s}_i = \frac{1}{s_{i\Sigma m}} \times \tilde{s}_i^{,}\,, \tag{37}$$

where:

$$s_{i\Sigma m} = \sum\nolimits_{i=1}^{n} s_{im}^{,}\,. \tag{38}$$

where: $s_{i\Sigma m}$ as a sum of modes of non-normalized fuzzy significance coefficients of objective i.

The scaling procedure presented by using the Eq. (37) and the condition (38) is a modified version of the well known Linear Transformation – Sum Method, which is adapted for use when significance coefficients are expressed by using triangular fuzzy numbers, and its use ensures the satisfaction of the following condition:

$$\sum\nolimits_{i=1}^{n} s_{im} = 1\,, \tag{39}$$

where s_{im} as a mode of the triangular fuzzy significance coefficient of objective i.

3.1.3. Normalize responses of alternatives

The next step in the proposed methodology, is to normalize fuzzy responses and construct a normalized fuzzy decision matrix, as follows:

$$\tilde{D}^{*} = [\tilde{x}_{ij}^{*}], \tag{40}$$

where $\tilde{x}_{ij}^{*}$ as a normalized fuzzy response of alternative j on objective i.

The MCDM methods uses a normalization procedure to transform responses of alternatives with different data measurement units into comparable dimensionless values, which are in the interval [0,1]. For normalization, the Eq. (41) was proposed:

$$\tilde{x}_{ij}^{*} = \left(\frac{x_{ijl}}{x_i^{+}}, \frac{x_{ijm}}{x_i^{+}}, \frac{x_{iju}}{x_i^{+}} \right), \tag{41}$$

where:

$$x_i^{+} = \left(\sum_{j=1}^{m} x_{iju}^{2} \right)^{\frac{1}{2}}. \tag{42}$$

3.2. The fuzzy extension of the ratio system approach of the MOORA method

In relation to the ordinary, the selection of the most appropriate alternative based on the Ratio system approach of the MOORA method, is more complex when significances of objectives or responses of alternatives are expressed using the triangular fuzzy numbers. Therefore, the proposed fuzzy extension will be presented using the following steps:

Stage 1: Determine fuzzy overall performance index, for each of considered alternatives;

Stage 2: Defuzzification, i.e. transform a fuzzy into a crisp overall performance index; and

Stage 3: Select the optimal/most appropriate/most desirable alternative.

Stage 1: ***Determine fuzzy overall performance index, for each considered alternative***. To expand the approach proposed in subsections 2.1. and 2.4, to determine the overall fuzzy performance index when fuzzy numbers are used, based on Eq. (17), the following equation has been proposed:

$$\tilde{y}_j^{*} = \tilde{y}_j^{+} - \tilde{y}_j^{-}, \tag{43}$$

where:

$$\tilde{y}_j^{+} = \sum_{i \in \Omega_{\max}} \tilde{s}_i \times \tilde{x}_{ij}^{*}; \text{ and} \tag{44}$$

$$\tilde{y}_j^{-} = \sum_{i \in \Omega_{\min}} \tilde{s}_i \times \tilde{x}_{ij}^{*}, \tag{45}$$

where: y_j^{+} as a sum of normalized responses of alternative j on objectives to be maximized; y_j^{-} as a sum of normalized responses of alternative j on objectives to be minimized; $\Omega_{\max}$ as a set of objectives to be maximized; and $\Omega_{\min}$ as a set of objectives to be minimized.

Results obtained by using the Eq. (43) are triangular fuzzy numbers. To enable evaluation and ranking of considered alternatives, these triangular fuzzy numbers is necessary to translate into a form suitable for ranking.

Stage 2: ***Defuzzify the fuzzy overall performance index into crisp overall performance index, for each alternative***. The conversion from a fuzzy overall into a crisp overall performance index can be performed by using any of methods described in subsection 1.3., which can be represented by the Eq. (46):

$$y_j^* = gm(\tilde{y}_j^*) . \tag{46}$$

If a decision maker has no risk-taking attitude, then the crisp overall performance indexes can be calculated using the equations (9) or (13). In contrast, if a decision maker has a risk-taking attitude then the Eq. (10) can be much more appropriate, because it enables the decision maker to assign greater significance to the mean of the fuzzy number.

Stage 3: ***Select the optimal alternative***. Based on the results obtained using the previous steps, alternatives can be ranked according to the descending order of y_j^*, and the one with the maximum value of y_j^* is the best, which is expressed by Eq. (20).

If two or more alternatives have the same rank, but decision maker still wants to determine which of them is most appropriate, Kaufmann and Gupta's method, can be used.

3.3. The fuzzy extension of the reference point approach of the MOORA method

Optimization procedure based on the Reference point approach, when fuzzy numbers are used, can be also divided into the following three stages, as follows:

Stage 1: Determine the fuzzy reference point;

Stage 2: Calculate the distances between fuzzy ratings and fuzzy reference point, for each alternative, and determine the maximum distances;

Stage 3: Select the optimal alternative.

Stage 1: ***Determine the fuzzy reference point.*** When responses of alternatives are expressed using a fuzzy numbers, the use of the fuzzy instead of ordinary reference point is suggested, as follows:

$$\tilde{R} = (\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n) , \tag{47}$$

where: $\tilde{r}_i$ as *i*th coordinate of the fuzzy reference point, and $\tilde{r}_i = (r_{il}, r_{im}, r_{iu})$.

The smallest possible value, the most promising value, and the largest possible value of *i*th coordinate of the fuzzy reference point are determined using the following equations:

$$r_{il} = \begin{cases} \max\limits_j x_{ijl}^*; & i \in \Omega_{\max} \\ \min\limits_j x_{ijl}^*; & i \in \Omega_{\min} \end{cases} ; \tag{48}$$

$$r_{im} = \begin{cases} \max\limits_j x_{ijm}^*; & i \in \Omega_{\max} \\ \min\limits_j x_{ijm}^*; & i \in \Omega_{\min} \end{cases} ; \tag{49}$$

$$r_{iu} = \begin{cases} \max\limits_j x_{iju}^*; & i \in \Omega_{\max} \\ \min\limits_j x_{iju}^*; & i \in \Omega_{\min} \end{cases} . \tag{50}$$

Stage 2: ***Calculate distances between fuzzy ratings and the fuzzy reference point, for each alternative, and determine the maximum distances.*** In this step it is necessary, for each alternative, to determine its maximal distance to the reference point.

However, because the normalized responses of alternatives and the coordinates of the reference point are fuzzy numbers, the obtained distances are fuzzy numbers, too. In order to provide the efficient but still easy to use, fuzzy Reference point approach, there have been proposed four approaches to determine maximum distances between alternatives and the fuzzy reference point.

The First approach. In the first approach, the distance d_{ij} between the fuzzy normalized response of alternative j on objective i and ith coordinate of the fuzzy reference point is defined as a distance between its generalized means, as follows:

$$d_{ij} = gm(\tilde{s}_i) \left| gm(\tilde{r}_i) - gm(\tilde{x}_{ij}^*) \right|. \tag{51}$$

where: d_{ij} as an unsigned distance of alternative j to the ith coordinate of the fuzzy reference point.

In the Eq. (51), fuzzy significance coefficients, normalized fuzzy responses and coordinates of the fuzzy reference point are transformed into appropriate crisp numbers. For the sake of transforming from fuzzy into crisp numbers, i.e. defuzzification, there can be used any of the procedures discussed in the subsection 1.3, but there have been proposed the use of the Eq. (13), because of its simplicity.

The Second approach. In the second approach to determine d_{ij} there have been proposed the proven Vertex method, as follows:

$$d_{ij} = gm(\tilde{s}_i)\, d_{vert}(\tilde{r}_i, \tilde{x}_{ij}^*). \tag{52}$$

The Third approach. In the third approach, the calculation of unsigned distance d_{ij} is based on the use of the Eq. (15), as shown below:

$$d_{ij} = gm(\tilde{s}_i)\ \ d_{\max}(\tilde{r}_i, \tilde{x}_{ij}^*). \tag{53}$$

In our opinion, this approach is very appropriate for use in the Reference point approach of the MOORA method.

The Fourth approach. Because the reference point approach of the ordinary MOORA method is based on the Min-Max metric, then instead of $gm(\tilde{s}_i)$ in the Eq. (53) $\tilde{s}_{iu}$ is used, which ensures that d_{ij} really represents the maximum distance of alternative j to the ith coordinate of the fuzzy reference point. After that, the Eq. (53) has the following form:

$$d_{ij} = \tilde{s}_{iu}\ \ d_{\max}(\tilde{r}_i, \tilde{x}_{ij}^*). \tag{54}$$

After the use of any of the above four approaches, the maximum unsigned distance of alternative j to the reference point can be determined as follows:

$$d_j^{\max} = \max_i d_{ij}. \tag{55}$$

Stage 3: ***Select the optimal alternative.*** After applying one of four proposed approaches and determining the maximum unsigned distance of any alternative to the reference point, the determination of the most appropriate alternative remains the same as in the ordinary MOORA method, and it is performed by using the Eq. (25).

4. Case study and discussion

To demonstrate the proposed extension of the MOORA method, its use in solving a particular problem has been shown in this section. Suppose that a Mining company XYZ from Serbia plans to start exploitation of a new mine with surface mining. Its geographical location, i.e. the distance of the new mine from the existing flotation, does not provide a cost-effective transportation of the excavated ore. Therefore, the team of experts was formed with the aim to evaluate the grinding circuit designs and propose the most appropriate one.

They consider three alternatives A_1, A_2, and A_3, i.e. three typical grinding circuit designs.

The first typical grinding circuit design is based on a two-stage grinding, where rod mills are used for primarily grinding and ball mills are used for a secondary grinding. It is also the most conventional design of all considered. It takes advantage of both types of mentioned mills, but also requires a greater amount of heavy equipment.

In contrast to the previously described, the second typical design uses a single-stage grinding process and ball mills.

Finally, the third grinding circuit design also uses a two-stage grinding process, but it also has its own characteristic, out of which the use of autogenous or semi-autogenous mills for primarily grinding is highlighted. These mills have a number of advantages, but there are some limitations on their use.

At the beginning of the evaluation, each expert evaluates the objectives, using the linguistic variables from Table 1. Assigned linguistic variables and the significances of objectives, obtained by using Eq. (35) and Eq. (37), are shown in Table 2.

Table 2. Significances of objectives

Criteria		E_1	E_2	E_3	$\tilde{s}_i'$	$\tilde{s}_i$
Grinding efficiency	C_1	VH	H	H	(6.67, 8.33, 9.33)	(0.19, 0.24, 0.27)
Economic efficiency	C_2	H	M	M	(4.00, 5.83, 7.67)	(0.11, 0.17, 0.22)
Technological reliability	C_3	VH	M	VH	(6.33, 8.33, 9.00)	(0.18, 0.24, 0.26)
Capital investment costs	C_4	M	H	VH	(5.67, 7.50, 8.67)	(0.16, 0.21, 0.25)
Envionmental impact	C_5	L	H	M	(3.33, 5.00, 6.67)	(0.10, 0.14, 0.19)

In the next step, experts evaluate the performance ratings of considered alternatives to the selected criteria, also by using linguistic variables from Table 1. Assigned linguistic variables for responses and corresponding quantitative values, obtained by using the Eq. (34), are shown in Table 3.

Table 3. Ratings of objectives

Criteria		E_1	E_2	E_3	$\tilde{x}_{ij}$
C_1	A_1	VH	H	H	(6.67, 8.33, 9.33)
C_1	A_2	H	M	M	(4.00, 5.83, 7.67)
C_1	A_3	VH	M	VH	(6.33, 8.33, 9.00)
C_2	A_1	M	M	M	(3.00, 5.00, 7.00)

Continued Table 3

Criteria		E_1	E_2	E_3	$\tilde{x}_{ij}$
C_2	A_2	H	M	M	(4.00, 5.83, 7.67)
C_2	A_3	VH	M	H	(5.67, 7.50, 8.67)
C_3	A_1	VH	H	VH	(7.33, 9.17, 9.67)
C_3	A_2	H	H	H	(6.00, 7.50, 9.00)
C_3	A_3	VH	H	VH	(7.33, 9.17, 9.67)
C_4	A_1	VH	H	H	(6.67, 8.33, 9.33)
C_4	A_2	VH	M	M	(4.67, 6.67, 8.00)
C_4	A_3	M	H	H	(5.00, 6.67, 8.33)
C_5	A_1	VH	M	VH	(6.33, 8.33, 9.00)
C_5	A_2	H	L	VH	(5.00, 6.58, 7.67)
C_5	A_3	M	L	H	(3.33, 4.92, 6.67)

Based on the data from Table2 and Table 3, the fuzzy decision matrix, shown in Table 4, was formed.

Table 4. Fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
$\tilde{s}_i$	(0.19, 0.24, 0.27)	(0.11, 0.17, 0.22)	(0.18, 0.24, 0.26)	(0.16, 0.21, 0.25)	(0.10, 0.14, 0.19)
Opt.	max	max	max	min	min
A_1	(6.67, 8.33, 9.33)	(3.00, 5.00, 7.00)	(7.33, 9.17, 9.67)	(6.67, 8.33, 9.33)	(6.33, 8.33, 9.00)
A_2	(4.00, 5.83, 7.67)	(4.00, 5.83, 7.67)	(6.00, 7.50, 9.00)	(4.67, 6.67, 8.00)	(5.00, 6.58, 7.67)
A_3	(6.33, 8.33, 9.00)	(5.67, 7.50, 8.67)	(7.33, 9.17, 9.67)	(5.00, 6.67, 8.33)	(3.33, 4.92, 6.67)

The norm x_i^+, for any objective, are determined using the Eq. (42) and these values are shown in Table 5.

Table 5. Normalization factors (norms)

	C_1	C_2	C_3	C_4	C_5
x_i^+	15.06	13.52	16.37	14.85	13.57

Based on data from Tables 4 and 5, using the Eq. (41), the normalized fuzzy decision matrix was formed. The normalized fuzzy decision matrix is shown in Table 6.

The Ratio system approach. Based on data from Table 6, the fuzzy performance rating obtained on the basis of objectives to be maximized $\tilde{y}_j^+$, for each alternative, was calculated by using the Eq. (44). On the basis of same data, the fuzzy performance rating obtained on objectives to be minimized $\tilde{y}_j^-$ was determined using the Eq. (45), for each alternative as well. These performance ratings are shown in columns I and II of Table 7.

Table 6. Normalized fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
$\tilde{s}_i$	(0.19, 0.24, 0.27)	(0.11, 0.17, 0.22)	(0.18, 0.24, 0.26)	(0.16, 0.21, 0.25)	(0.10, 0.14, 0.19)
Opt.	max	max	max	min	min
A_1	(0.44, 0.55, 0.62)	(0.22, 0.37, 0.52)	(0.45, 0.56, 0.59)	(0.45, 0.56, 0.63)	(0.47, 0.61, 0.66)
A_2	(0.27, 0.39, 0.51)	(0.30, 0.43, 0.57)	(0.37, 0.46, 0.55)	(0.31, 0.45, 0.54)	(0.37, 0.49, 0.56)
A_3	(0.42, 0.55, 0.60)	(0.42, 0.55, 0.64)	(0.45, 0.56, 0.59)	(0.34, 0.45, 0.56)	(0.25, 0.36, 0.49)

Table 7. The overall fuzzy performance indexes

	$\tilde{y}_j^+$	$\tilde{y}_j^-$	$\tilde{y}_j^*$
	I	II	III
A_1	(0.19, 0.33, 0.43)	(0.12, 0.21, 0.28)	(-0.09, 0.12, 0.31)
A_2	(0.15, 0.27, 0.4)	(0.09, 0.17, 0.24)	(-0.09, 0.11, 0.32)
A_3	(0.21, 0.36, 0.45)	(0.08, 0.15, 0.23)	(-0.02, 0.21, 0.37)

The overall performance indexes of considered alternatives, calculated using the Eq. (43), are shown in column III of Table 7.

Finally, by using the Eq. (10), or the Eq. (11), and different values for the parameter λ, the decision maker can determine the ranking order of alternatives, and select the most appropriate one. Also, by using different values of the coefficient λ, decision makers can consider different scenarios, such as pessimistic, moderate and optimistic.

The ranking results obtained by using the Eq. (10) and some characteristic values of coefficient λ, are shown in Table 8.

Table 8. Ranking results obtained for characteristic values of λ

		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
	$\tilde{y}_j^*$	y_j^*	Rank	y_j^*	Rank	y_j^*	Rank
A_1	(–0.09, 0.12, 0.31)	0.014	2	0.115	2	0.216	3
A_2	(–0.09, 0.11, 0.32)	0.009	3	0.110	3	0.212	2
A_3	(–0.02, 0.21, 0.37)	0.093	1	0.192	1	0.292	1

According to the opinion of experts, who were involved in the evaluation, the ranking orders shown in Table 8 are correct, although it is a bit surprising because that the dominance of alternative A_3 is very obvious.

The transformation from the fuzzy into crisp overall performance indexes can be made using any of methods discussed in subsection 1.3, as well as many other defuzzification methods, which are not considered in this paper.

Results obtained by defuzzification methods discussed in subsection 1.3, are shown in Table 9.

Table 9. Ranking results obtained by using different defuzzification methods

		Kaufmann and Gupta		Chiu and Park λ = 0		Liou and Wang λ = 0.5		Opricovic and Tzeng	
	$\tilde{y}_j^*$	y_j^*	Rank	y_j^*	Rank	y_j^*	Rank	y_j^*	Rank
A_1	(–0.09, 0.12, 0.31)	–0.004	2	0.173	2	0.115	2	0.114	2
A_2	(–0.09, 0.11, 0.32)	–0.006	3	0.165	3	0.110	3	0.111	3
A_3	(–0.02, 0.21, 0.37)	0.033	1	0.291	1	0.192	1	0.187	1

In this case, all methods have the same ranking order of alternatives, however, many authors warn that different defuzzification methods may give different results.

In the considered example, the symmetrical TFNs have been used. A more realistic consideration of applied defuzzification methods for obtaining ranking orders is shown in appendix A, where the example with non-symmetrical fuzzy numbers is also discussed.

The Reference point approach. The pwrocess of determining the most appropriate alternative based on the Reference point approach, is started from Table 6. Using data from Table 6 and Eq. (47), or more precisely equations (48), (49) and (50), coordinates of the fuzzy reference point are determined.

Table 10 shows the coordinates of the fuzzy reference point.

Table 10. Fuzzy reference point

	C_1	C_2	C_3	C_4	C_5
$\tilde{R}$	(0.44, 0.55, 0.62)	(0.42, 0.55, 0.64)	(0.45, 0.56, 0.59)	(0.31, 0.45, 0.54)	(0.25, 0.36, 0.49)

Based on the data from Tables 10 and 6, using the first approach proposed in Section 4.3, i.e. using Eq. (52), distances between the considered alternative and the fuzzy reference point, are calculated. Mentioned distances, the maximum distances between alternatives and reference point and ranking order of alternatives, are given in Table 11.

Table 11. Distances and rank of alternatives based on the first fuzzy Reference point approach

	C_1	C_2	C_3	C_4	C_5	*max*	*Rank*
A_1	0.00	0.03	0.00	0.02	0.03	0.03	2
A_2	0.04	0.02	0.02	0.00	0.02	0.04	3
A_3	0.00	0.00	0.00	0.00	0.00	0.00	1

Based on data shown in Table 12, it can be seen that, by using the Ratio system approach of the MOORA method, the best ranked alternative is also the alternative A_3.

Distances of considered alterantives to the reference point and the rank of alternatives, obtained using the second, third and fourth approaches, are shown in Tables 12, 13 and 14.

Table 12. Distances and rank of alternatives based on the second fuzzy Reference point approach

	C_1	C_2	C_3	C_4	C_5	*max*	*Rank*
A_1	0.00	0.03	0.00	0.02	0.03	0.03	2
A_2	0.04	0.02	0.02	0.00	0.02	0.04	3
A_3	0.00	0.00	0.00	0.00	0.00	0.00	1

Table 13. Distances and rank of alternatives based on the third fuzzy Reference point approach

	C_1	C_2	C_3	C_4	C_5	*max*	*Rank*
A_1	0.00	0.03	0.00	0.02	0.03	0.04	2
A_2	0.04	0.02	0.02	0.00	0.02	0.04	2
A_3	0.00	0.00	0.00	0.00	0.00	0.00	1

Table 14. Distances and rank of alternatives based on the fourth fuzzy Reference point approach

	C_1	C_2	C_3	C_4	C_5	*max*	*Rank*
A_1	0.00	0.04	0.00	0.03	0.05	0.05	2
A_2	0.05	0.03	0.03	0.00	0.02	0.05	2
A_3	0.01	0.00	0.00	0.01	0.00	0.01	1

Using a more precise calculation, i.e. by using a larger number of decimal places, a different ranking order of alternatives is obtained, which is shown in Table 15.

Table 15. Distances and rank of alternatives based on the fourth fuzzy Reference point approach

	C_1	C_2	C_3	C_4	C_5	*max*	*Rank*
A_1	0.000	0.043	0.000	0.033	0.048	0.048	3
A_2	0.047	0.027	0.026	0.000	0.023	0.047	2
A_3	0.006	0.000	0.000	0.006	0.000	0.006	1

However, more precise calculation, based on the third proposed Reference point approach has not caused any changes in the rank order of alternatives. This confirms that the use of different approaches may have an influence on the rank order of alternatives and that the selection of fuzzy Reference point approach is also very important, as shown in appendix B.

Conclusion

In order to rank alternatives in fuzzy environment, MCDM methods must be able to perform ranking based on overall fuzzy responses, or must transform overall fuzzy responses into crisp responses before they perform ranking. Also, greater benefits from the use of fuzzy numbers can be obtained if the defuzzification is done in the later stages of MCDM methods.

In order to enable the use of MOORA method in fuzzy environments, in this paper, we have proposed an extension which is based on the transformation of overall fuzzy responses into exact values, before ranking.

This paper discusses the use of several methods for defuzzification and several methods for calculation the distance between two fuzzy numbers. These methods allow the use of the Ratio system and the Reference point approach of the MOORA method in fuzzy environment, as in the considered example is confirmed.

Methods offered for defuzzification and methods offered to calculate the distance between two fuzzy numbers have their own specificities and advantages. However, in this paper no preference is given to any of them. Our goal has been to provide more opportunities because we believe that the proposed fuzzy extension of the MOORA method can be used as a basis for future researches, and that their authors may, depending on the problem being solved, choose the most appropriate one.

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APPENDIX A: CONSIDERATION OF THE EFFECTS OBTAINED BY USING A DIFFERENT DEFUZZIFICATION IN THE RATIO SYSTEM APPROACH OF THE MOORA METHOD

In this section, the effects of using various defuzzification methods on the ranking order of alternatives in Ratio system approach of the MOORA method have been discussed.

Suppose two fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ with equal upper and lower bounds, but the fuzzy number $\widetilde{B}$ has a slightly smaller mode, as shown in Figure A1.

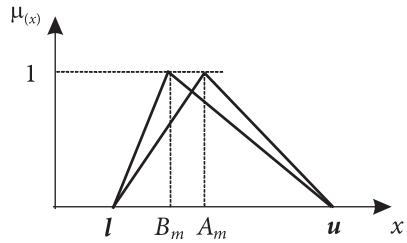

Fig. A1. The relationships between two TFNs.

In this case, it is completely obvious that the fuzzy number $\tilde{A}$ is greater than the fuzzy number $\tilde{B}$.

Table A1 shows four fuzzy numbers $\tilde{A}_1$, $\tilde{B}_1$, $\tilde{B}_2$ and $\tilde{B}_3$, and also there ranking orders obtained by using defuzzification methods discussed in the subsection 1.3.

Table A1. Ranking results obtained by using different defuzzification methods

		Kaufmann and Gupta		Chiu and Park λ = 0		Liou and Wang λ = 0.5		Opricovic and Tzeng	
		S_j	Rank	S_j	Rank	S_j	Rank	S_j	Rank
$\tilde{A}_1$	(0.3, 0.5, 0.7)	0.500	1	0.500	1	0.500	1	0.500	1
$\tilde{B}_1$	(0.3, 0.4, 0.7)	0.450	2	0.467	2	0.450	2	0.467	2
$\tilde{B}_2$	(0.2, 0.4, 0.7)	0.425	3	0.433	3	0.425	3	0.433	3
$\tilde{B}_3$	(0.1, 0.4, 0.7)	0.400	4	0.400	4	0.400	4	0.400	4

Fuzzy numbers $\tilde{A}_1$ and $\tilde{B}_1$ have same lower and upper bounds, but the $\tilde{A}_1$ has a higher mode, and therefore it is also ranked higher. Fuzzy numbers $\tilde{B}_2$ and $\tilde{B}_3$ have the same mode and upper limits as a fuzzy number $\tilde{B}_1$, but their lower bounds are less which also has an effect to their ranks.

In the considered example, all defuzzification methods gave the same ranking order of alternatives. However, the increase of the lower bound of a triangular fuzzy number $\tilde{B}$ from l to $B_{l''}$, or a variation of its upper bound between $B_{u'}$ and $B_{u''}$; makes the ranking of fuzzy numbers $\tilde{A}$ and $\tilde{B}$ much more complex, as it is shown in Figure A2.

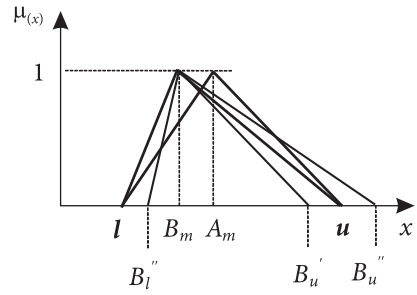

Fig. A2. The relationships between two TFNs.

The changes of upper limits of fuzzy numbers $\tilde{B}_1$, $\tilde{B}_2$ and $\tilde{B}_3$, from 0.7 to 0.8, in Table A2, and to in 0.9, in Table A3, have a significant effect on their ranking orders, as can be seen in Tables A2 and A3.

Table A2. Ranking results obtained by using different defuzzification methods

		Kaufmann and Gupta		Chiu and Park λ = 0		Liou and Wang λ = 0.5		Opricovic and Tzeng	
		S_i	Rank	S_i	Rank	S_i	Rank	S_i	Rank
$\tilde{A}_1$	(0.3, 0.5, 0.7)	0.500	1	0.500	1	0.500	1	0.500	1
$\tilde{B}_1$	(0.3, 0.4, 0.8)	0.475	2	0.500	1	0.475	2	0.500	1
$\tilde{B}_2$	(0.2, 0.4, 0.8)	0.450	3	0.467	3	0.450	3	0.467	3
$\tilde{B}_3$	(0.1, 0.4, 0.8)	0.425	4	0.433	4	0.425	4	0.433	4

From Tables A2 and A3, it can be concluded that different defuzzification methods give the different ranking orders.

Table A3. Ranking results obtained by using different defuzzification methods

		Kaufmann and Gupta		Chiu and Park λ = 0		Liou and Wang λ = 0.5		Opricovic and Tzeng	
		S_i	Rank	S_i	Rank	S_i	Rank	S_i	Rank
$\tilde{A}_1$	(0.3, 0.5, 0.7)	0.500	1	0.500	2	0.500	1	0.500	2
$\tilde{B}_1$	(0.3, 0.4, 0.9)	0.500	1	0.533	1	0.500	1	0.533	1
$\tilde{B}_2$	(0.2, 0.4, 0.9)	0.475	3	0.500	2	0.475	3	0.500	2
$\tilde{B}_3$	(0.1, 0.4, 0.9)	0.450	4	0.467	4	0.450	4	0.467	4

APPENDIX B: HOW TO DETERMINE THE DISTANCE IN THE FUZZY REFERENCE POINT APPROACH OF THE MOORA METHOD

In this section, the effects of methods proposed for determining the distance between normalized fuzzy responses and the reference point are considered.

Suppose four alternatives, denoted as A_1, A_2, A_3, A_4 , whose fuzzy responses are shown in Table B1. In this table there are shown some elements relevant to determine the distances between the alternatives and the reference point.

Table B1. Data for selecting the optimal alternative based on the reference point approach

		gm_j	$r_l - a_{jl}$	$r_m - a_{jm}$	$r_u - a_{ju}$
A_1	(0.3, 0.5, 0.7)	0.500	0.10	0.00	0.10
A_2	(0.4, 0.5, 0.6)	0.500	0.00	0.00	0.20
A_3	(0.3, 0.5, 0.6)	0.467	0.10	0.00	0.20
A_4	(0.3, 0.4, 0.8)	0.500	0.10	0.10	0.00
$\tilde{R}$	(0.4, 0.5, 0.8)	0.567			

In Table B2, there are shown distances between the alternatives and the fuzzy reference point obtained by using three approaches discussed in the subsection 3.3. As it can be seen, the proposed approaches give different ranking results.

Table B2. Ranking results obtained by using approaches proposed in the subsection 3.3

		1st approach		2nd approach		3rd approach	
		d_j	Rank	d_j	Rank	d_j	Rank
A_1	(0.3, 0.5, 0.7)	0.067	1	0.082	1	0.100	1
A_2	(0.4, 0.5, 0.6)	0.067	1	0.115	3	0.200	3
A_3	(0.3, 0.5, 0.6)	0.100	4	0.129	4	0.200	3
A_4	(0.3, 0.4, 0.8)	0.067	1	0.082	1	0.100	1

To compare the effects obtained by using different approaches, in Table B3 are shown results obtained by applying the third approach, but different reference points. In this case, except for the fuzzy reference point, the ordinary reference point was used, which is determined by using Eq. (56):

$$r_i = \max_i \; x^*_{iju}, \tag{56}$$

when $i \in \Omega_{\max}$.

As it can be seen from Table B3, the obtained ranking orders of considered alternatives are also different.

Table B3. Ranking results obtained by using different approaches and different reference points

		d_{max} Fuzzy		d_{max} Crisp	
		d_j	Rank	d_j	Rank
A_1	(0.3, 0.5, 0.7)	0.100	1	0.90	2
A_2	(0.4, 0.5, 0.6)	0.200	3	0.90	2
A_3	(0.3, 0.5, 0.6)	0.200	3	1.00	4
A_4	(0.3, 0.4, 0.8)	0.100	1	0.90	1

Dragisa STANUJKIC was born in Serbia in 1964. He received his BSc degree in Mechanical Engineering from the Technical Faculty in Bor in 1988. He received his MSc degree in Information Systems in 1993, and his PhD in 2008, from the University of Belgrade. He is currently an Associate Professor at the Department of Information Technology at the Faculty of Management of Megatrend University in Belgrade. His research interests are decision making theory, expert systems and intelligent decision support systems.