

Supplementary material

**MULTI-OBJECTIVE SUSTAINABILITY OPTIMIZATION OF CCHP SYSTEMS
CONSIDERING THE DISCRETENESS OF EQUIPMENT CAPABILITIES**

Xusheng REN, Shimin DING, Lichun DONG, Lixiao QIN

S1. Fuzzy pairwise comparison method

The fuzzy pairwise comparison method to determine the weights has the following five steps based on the work of Chang (1996):

Step 1: Establishing a comparison matrix.

Firstly, a comparison matrix(M1) using linguistic terms can be established as shown in Table S1, then transforming the comparison matrix M1 into M2 expressed by triangular fuzzy numbers using the scales presented in Table S1, as shown below.

$$M2 = \begin{matrix} & O_1 & O_2 & \cdots & O_n \\ O_1 & \tilde{1} & \tilde{m}_{12} & \cdots & \tilde{m}_{1n} \\ O_2 & \tilde{m}_{21} & \tilde{m}_{22} & \cdots & \tilde{m}_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_n & \tilde{m}_{n1} & \tilde{m}_{n2} & \cdots & \tilde{m}_{nn} \end{matrix} \quad (S1)$$

$$\tilde{m}_{ij} = [\tilde{m}_{ij}^L, \tilde{m}_{ij}^M, \tilde{m}_{ij}^U], \quad \tilde{m}_{ji} = 1 / \tilde{m}_{ij}, \quad i, j = 1, 2, \dots, n.$$

Where O_n is the n -th objective; \tilde{m}_{ij} is a triangular fuzzy number representing the relative importance of the i -th criterion compared with the j -th criterion. For example, if the relative importance of O_1 compared with O_2 is "Fairly strong priority", we can express it by FS in matrix M1 and a fuzzy number (3/2,2,5/2) in matrix M2. As an illustrative example, the following two matrices M1 and M2 are listed below.

$$M1 = \begin{matrix} & ATC & Eco & PEC \\ ATC & JE & FS & FS \\ Eco & RFS & JE & E \\ PEC & RFS & RE & JE \end{matrix}$$

$$M2 = \begin{matrix} & ATC & Eco & PEC \\ ATC & (1,1,1) & (3/2,2,5/2) & (3/2,2,5/2) \\ Eco & (2/5,1/2,2/3) & (1,1,1) & (2/3,1,3/2) \\ PEC & (2/5,1/2,2/3) & (2/3,1,3/2) & (1,1,1) \end{matrix}$$

Table S1. The linguistic terms and the corresponding fuzzy numbers for pairwise comparison (Chang, 1996)

Linguistic terms	Abbreviations	Fuzzy scales
Just equal	JE	(1,1,1)
Equal priority	E	(2/3,1,3/2)
Weak priority	W	(1,3/2,2)
Fairly strong priority	FS	(3/2,2,5/2)
Very strong priority	VS	(2,5/2,3)
Absolute priority	A	(5/2,3,7/2)
Reciprocals	RE, RW, RFS, RVS, RA	The reciprocals of these fuzzy number

Step 2: Calculating the value of fuzzy synthetic extent with respect to the i -th objective.

The value of fuzzy synthetic extent can be calculated by Eqs (S2)–(S4), as shown below. $S_i = (S_i^L, S_i^M, S_i^U)$ is a triangular fuzzy number that represents the value of the fuzzy synthetic extent with respect to the i -th objective.

$$S_i = \sum_{j=1}^m \tilde{m}_{ij} \otimes \left[\sum_{i=1}^n \sum_{j=1}^m \tilde{m}_{ij} \right]^{-1}, \quad (S2)$$

where

$$\sum_{j=1}^m \tilde{m}_{ij} = \left(\sum_{j=1}^m \tilde{m}_{ij}^L, \sum_{j=1}^m \tilde{m}_{ij}^M, \sum_{j=1}^m \tilde{m}_{ij}^U \right), \quad i = 1, 2, \dots, n; \quad (S3)$$

$$\left[\sum_{i=1}^n \sum_{j=1}^m \tilde{m}_{ij} \right]^{-1} = \left(\frac{1}{\sum_{i=1}^n \sum_{j=1}^m \tilde{m}_{ij}^U}, \frac{1}{\sum_{i=1}^n \sum_{j=1}^m \tilde{m}_{ij}^M}, \frac{1}{\sum_{i=1}^n \sum_{j=1}^m \tilde{m}_{ij}^L} \right). \quad (S4)$$

According to the above equations, the fuzzy synthetic extent of ATC can be obtained as follows:

$$S_1 = \left[(1,1,1) + (3/2,2,5/2) + (3/2,2,5/2) \right] \otimes \left[\begin{array}{c} (1,1,1) + (3/2,2,5/2) + (3/2,2,5/2) \\ + (2/5,1/2,2/3) + (1,1,1) + (2/3,1,3/2) \\ + (2/5,1/2,2/3) + (2/3,1,3/2) + (1,1,1) \end{array} \right]^{-1} = (4,5,6) \otimes (0.081,0.100,0.123) = (0.324,0.500,0.738).$$

The fuzzy synthetic extent of eco-costs and PEC can be obtained in a similar way, $S_2 = (0.168,0.250,0.389)$, $S_3 = (0.168,0.250,0.389)$.

Step 3: Determining the possibility matrix.

The elements \tilde{p}_{ij} of possibility matrix can be determined by the Eq. (S5). Here \tilde{p}_{ij} represents the degree of possibility of the triangular fuzzy number $S_i \geq S_j$.

$$\tilde{p}_{ij} = V(S_i > S_j) = \begin{cases} 1 & \text{if } S_i^M - S_j^U \\ 0 & \text{if } S_j^L - S_i^U \\ \frac{S_j^L - S_i^U}{(S_i^M - S_i^U) + (S_j^M - S_j^L)} & \text{Otherwise} \end{cases}. \quad (S5)$$

Thus, because of $S_{ATC}^M \geq S_{Eco}^M$; and $\tilde{p}_{21} =$ because S_{ATC} and S_{Eco} do not satisfy $S_{Eco}^M \geq S_{ATC}^M$ and $S_{ATC}^L \geq S_{Eco}^U$.

$$M2 = \begin{array}{ccc|ccc} & ATC & Eco & PEC & & \\ ATC & / & 1 & 1 & & \\ Eco & 0.206 & / & 1 & & \\ PEC & 0.206 & 1 & / & & \end{array}$$

Step 4: Calculating the degree of possible for the fuzzy synthetic extent with respect to each objective to be greater than that with respect to all of the other objectives.

It can be determined by Eq. (S6). $d'(O_i)$ means the weight of the i -th objective.

$$d'(O_i) = \min V(S_i \geq S_k) \text{ for } k = 1, 2, \dots, n \text{ and } k \neq i.$$

(S6) Thus, the weight of objective eco-costs can be determined as below.

$$d'(O_{Eco}) = \min[V(S_2 \geq S_1), V(S_2 \geq S_3)] = 0.206.$$

In the same way, the weights of ATC, eco-costs and PEC can be obtained as

$$W' = (d'(O_{ATC}), d'(O_{Eco}), d'(O_{PEC}))^T = (1, 0.206, 0.206)^T.$$

Step 5: Normalizing weights.

We can get the normalized weights from Eq. (S7). In the equation, $d(O_i)$ is the normalized weight of i -th objective.

$$d(O_i) = d'(O_i) / \left(\sum_{i=1}^n d'(O_i) \right).$$

(S7) And the normalized weights of ATC, eco-costs and PEC are as follows:

$$W = (0.708, 0.146, 0.146)^T.$$

S2. Assumptions to obtain load demands and solar irradiation

In this paper, in order to verify the validity of model, a hypothetical office building locating in Shanghai, Eastern China is established in EnergyPlus energy simulation software to acquire the load demands and solar irradiation. Shanghai is a subtropical monsoon climate, the annual average temperature in the city was 17.6 °C and the sunshine duration was 1,885.9 hours. And the hypothetical office building is a 25-storey building with a land square of 20000 m² with the following assumptions: 1) The entire office building is seen as one thermal zone; 2) While thermostat are used to maintain a comfortable temperature; 3) the electric equipment, lights, people densities and occupation are defined according to the public energy saving design standard.

At last, time aggregation methods (Marquant et al., 2017) which is an effective method to reduce the dimensionality of the problem and improve computational efficiency was used in this paper. And three typical days representing winter, mid-season and summer, respectively, which are used to reflect the seasonal and daily fluctuation of load demands in the building, were selected to reduce the total number of variables in the model. Noteworthy, the durations of each typical days are 120, 120 and 125 days, respectively, and each typical day is divided into 24 time periods with an hour between adjacent ones.

References

- Chang, D.-Y. (1996). Applications of the extent analysis method on fuzzy AHP. *Journal of Operational Research*, 95(3), 649–655. [https://doi.org/10.1016/0377-2217\(95\)00300-2](https://doi.org/10.1016/0377-2217(95)00300-2)
- Marquant, J. F., Evins, R., Bollinger, L. A., & Carmeliet, J. (2017). A holarchic approach for multi-scale distributed energy system optimisation. *Applied Energy*, 208, 935–953. <https://doi.org/10.1016/j.apenergy.2017.09.057>