Statyba

ISSN: 1392-1525 (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tcem19

A SIMULATION APPROACH TO RELIABILITY ASSESSMENT OF PLASTIC FRAMES

E. R. Vaidogas

To cite this article: E. R. Vaidogas (1997) A SIMULATION APPROACH TO RELIABILITY ASSESSMENT OF PLASTIC FRAMES, Statyba, 3:12, 28-37, DOI: 10.1080/13921525.1997.10531364

To link to this article: https://doi.org/10.1080/13921525.1997.10531364

Published online: 26 Jul 2012.

A SIMULATION APPROACH TO RELIABILITY ASSESSMENT OF PLASTIC FRAMES

E.R. Vaidogas

1. Introduction

The fundamental reliability analysis problem is to assess the structural (or system) reliability that may significantly differ from the individual structural component reliabilities. The redundancy or reserve strength of a structure implies that the reliability with respect to system failure is higher than the reliability with respect to component failure. Probabilistic analysis methods intended for an assessment of the relative high system reliability or, conversely, relative small probability of system failure are therefore combined with mechanical models that describe a structure as a system [1].

The rigid ideal plastic frame structures have been intensively studied within probabilistic analysis of structural systems and numerous studies have been reported (see the reviews in Refs [2-5]). Two main approaches to the assessment of the reliability with respect to the formation of a plastic mechanism are based on the theorems of limit analysis called the static theorem of admissible stress fields (lower bound theorem) and kinematic theorem of mechanisms (upper bound theorem). The methods developed within the both approaches are used to assess the reliability in terms of a lower and upper bound, respectively. It is generally agreed that the methods of the conservative reliability assessment are more relevant for engineering decision.

In the lower bound methods, the reliability problem is stated in the space of basic variables, and the limit state functions are derived from the formulations of the limit analysis theorems. The reliability or, conversely, the probability of plastic (or collapse) failure is then estimated using analytical procedures, such as first-order and second-order reliability methods or simulation-based reliability methods.

Due to the high order of reliability characteristic for structural systems, such simulation-based methods as directional simulation and importance sampling (both belonging to the so-called variance reduction techniques) have proved to be particularly suitable to the estimation of the probability of plastic failure [2,6]. The directional simulation may be applied when the reliability problem is stated in the standard Gaussian space. A transformation of the vector of basic variables to the standard Gaussian space is for most realistic problems usually nontrivial [7]. The transformation of safety margins of a rigid ideal plastic frame also involves some difficulties [2].

A separate group of methods intended for the estimation of the plastic failure probability has its base in the load-space formulation of the reliability problem [8-11]. The dimension of the load-space is in most realistic problems low in comparison with the dimension of the space of basic variable. It may be advantageous for a necessary integration in the load-space. The estimation of the plastic failure probability in the load-space is performed using the proportional loading approach. A problem resulting from the reliability formulation in the load-space is that the limit state functions have a randomnised form. Loading proportions therewith correspond to random and, in general case, individual resistances.

In this paper a method for the estimation of the collapse probability of rigid ideal plastic frames is suggested. The method combines features of directional simulation and radial sampling with the analysis of the frames using the proportional loading approach. The discretisation of the load space is replaced by random choice of the loading proportions. The analysis of a frame for each simulated proportion is performed on the bases of static or kinematic formulation of the limit equilibrium problem. These formulations are expressed as a dual pair of linear programming (LP) problems. A connecting link between the radial sampling procedure and the limit equilibrium problem is the load direction and distribution vector appearing in the formulations of the problem. This vector may be expressed and simulated as a radial direction in the load-space.

2. Discretisation of the load-spaceand radial sampling

The simplest way of the estimation of the plastic failure is to discretise the load-space in subspaces, each being characterized by one proportion of loads. The failure probability is then estimated for each proportion, and the overall failure probability is obtained through the summation over all proportions [8,9]. The mean and variance of a random frame resistance corresponding to an individual load proportion are approximated using an incremental loading model [1,7].

The approach „discretisation" plus „approximation of moments of the resistance distribution" has obvious shortcomings. Firstly, an increase in the dimension of the load-space causes a nonlinear increase in the number of loading proportions to be analyzed (see a numerical illustration in Ref [10]). Secondly, the mean and variance of the frame resistance corresponding to the considered loading proportion are approximated assuming that only one mode of the plastic failure identified for mean values of plastic moment capacities is possible. This is not always the case. Finally, the probability distribution of the resistance is assumed only on the basis of the two approximated distribution parameters. It is stated that the exact form of the probability distribution is not critical because the variability in the resistance is generally much less than that in the loads [9,10]. However, the variability in the resistance tends to increase with the complexity of the frame, more exactly, with the number of weakly correlated plastic moment capacities. A simple criterion for answering the question whether the variability in the resistance is small enough to neglect the distribution type problem is not given.

Several attempts to remove the above-mentioned problems were reported. The dimension of the load-space may be reduced by a reformulating of the initial probability estimation problem [10]. For example, only load combinations with a high likelihood of causing failure are used, or else two or more independent loads are assumed to be fully correlated. In such a manner the estimation problem is simplified even if with the loss in generality.

An alternative method of the discretisation of the load-space consists in a grouping of the loading proportions corresponding to the same mode of failure [11]. The method consists in the Monte Carlo analysis of a LP problem called the kinematic formulation of the limit equilibrium problem [12,13]. The LP problem is repeatedly solved for different observations of basic variables obtained through Monte Carlo simulation. The solving results are grouped according to the mode of failure, and the probability distribution of the resistance is statistically fitted for each mode from the computed values of the load magnitude (critical load factor). The overall plastic failure probability is then the sum of all failure probabilities corresponding to the failure modes that were observed during the Monte Carlo analysis. Consequently, the fitted resistance distribution is taken as representative for a segment of the load-space associated with an individual failure mode.

The combination „simulation plus LP" may be applied to the estimation of the plastic failure without consideration of the individual failure modes and discretisation of the load-space. The loading in the kinematic and static formulations of the limit equilibrium problem is expressed as a product of the unknown load magnitude and a predetermined load direction and distribution vector [13]. This vector defines the loading proportion, and it may be expressed as a unit directional vector. The kinematic and static formulations necessarily results in the same maximal load magnitude (collapse load). The plastic failure probability may be computed by integrating over all directions in the load space. The probability integral may be evaluated through the simulation that consists in the random choice of the directions. Such a evaluation lies at the basis of the directional simulation [3,7]. It is performed not in the standard Gaussian space but in the load-space, and therefore the evaluation procedure is referred as the radial sampling [14].

3. Mechanical model

Consider a plane frame of known configuration characterized by the vector $\boldsymbol{m}_0 \in \boldsymbol{R}^n$ which components are plastic moment capacities in n predetermined critical sections. The frame is acted upon by a set of concentrated loads represented by the vector $\boldsymbol{l} \in \boldsymbol{R}^m$ which is expressible as a product of the load magnitude λ and the load direction and distribution vector $\boldsymbol{a}$. The difference $n-m$ is the number of degrees of freedom. The load-bearing capacity of the frame for a given $\boldsymbol{a}$ is expressed in terms of the maximal load magnitude λ_0. It may be found by solving the static or kinematic formulations of the limit equilibrium problem. The static formulation has the form

$$max\left\{\lambda \mid \mathbf{E}\boldsymbol{m} \le \boldsymbol{m}_0,\ -\mathbf{E}\boldsymbol{m} \le \boldsymbol{m}_0,\ \mathbf{A}\boldsymbol{m} - \lambda\boldsymbol{\alpha} = \boldsymbol{0}\right\} \quad (1)$$

where E is the unit matrix; $\boldsymbol{m}$ is the vector of bending moments; A is the matrix of equilibrium equations, and $\boldsymbol{0}$ is the zero-vector.

If the plastic moment capacities and loads are taken as random and represented by the respective random vectors $\boldsymbol{M}_0$ and $\boldsymbol{L}$, the load magnitude λ and the load-bearing capacity λ_0 are stochastized for a given $\boldsymbol{a}$. One speaks in this case about plastic failure probability conditioned on the direction $\boldsymbol{a}$ in the load space $\boldsymbol{R}^m$ [7, 11]. The ideas of the radial sampling in the load-space can in this case be applied to estimate the plastic failure probability.

4. Estimation of the plastic failure probability by radial sampling in the load-space

The probability of plastic failure, p_f, may be expressed in the Cartesian coordinates as

$$p_f = \int_{R^{n+m}} 1(x, y)\, f_{M0}(x) f_L(y)\, dx\, dy \quad (2)$$

where $f_{M0}()$ and $f_L()$ are the joint probability density function (PDF) of the vector of limiting moments and the load vector, respectively; $1()$ is an indicator function defined by

$$1(\boldsymbol{m}_0, \boldsymbol{l}) = \begin{cases} 1 & \text{if } r(\boldsymbol{m}_0,\alpha) \le s(\boldsymbol{l}) \\ 0 & \text{if } r(\boldsymbol{m}_0,\alpha) > s(\boldsymbol{l}) \end{cases} \quad (3)$$

Here the scalar

$$s = s(\boldsymbol{l}) = \|\boldsymbol{l}\| = \lambda \quad (4)$$

is a norm of the load vector value $\boldsymbol{l}$, and the scalar

$$r(\boldsymbol{m}_0,\boldsymbol{\alpha}) = \lambda_0 \quad (5)$$

is computed by solving the LP problem (1) for the value of the random vector of limiting moments, $\boldsymbol{m}_0$, and the load direction and distribution vector

$$\boldsymbol{\alpha} = \boldsymbol{l}/\|\boldsymbol{l}\| \quad (6)$$

The simple Monte Carlo estimate of the failure probability p_f in this case has the form

$$\hat{p}_{f1,N} = N^{-1} \sum_{j=1}^{N} 1(\boldsymbol{m}_{0j}, \boldsymbol{l}_j) \quad (7)$$

where $\boldsymbol{m}_{0j}$ and $\boldsymbol{l}_j$ are observations of $\boldsymbol{M}_0$ and $\boldsymbol{L}$.

The random external loading $\boldsymbol{L}$ in this case is represented as product of the magnitude variable

$$S = \|\boldsymbol{L}\| \quad (8)$$

and the direction and distribution vector

$$\boldsymbol{A} = \boldsymbol{L}/\|\boldsymbol{L}\| \quad (9)$$

This fits into the logic pattern of the reliability estimation by the directional sampling and, first of all, radial sampling in the load-space [7,6,14,15]. The random directional unit vector $\boldsymbol{A}$ is, in general, nonuniformly distributed on the m-dimensional unit (hyper)sphere Ω_m in $\boldsymbol{R}^m$. If the random external loading $\boldsymbol{L}$ is defined only on the positive subspace $\boldsymbol{R}^+$of $\boldsymbol{R}^m$, as for instance, the loads are nonalternating, the vector $\boldsymbol{A}$ is nonuniformly distributed on the spherical segment

$$\omega_m = \Omega_m \cap \boldsymbol{R}^+$$

It is possible to rewrite (2) in (hyper-) polar coordinate system as

$$p_f = \int_{\alpha \in \omega_m} \left[\int_0^{\infty} \left(1 - F_S(s|\alpha)\right) f_R(s|\alpha) ds \right] f_A(\alpha)\, d\alpha$$

$$= \int_{\alpha \in \omega_m} p_{f|A}(\alpha) f_A(\alpha)\, d\alpha \quad (10)$$

where $F_S(.|.)$ is the conditional cumulative distribution function (CDF) of the magnitude of the load vector, S; $f_A()$ is the PDF of the random unit vector A; $f_R(.|.)$ is the conditional PDF of the frame resistance for a given radial direction $\boldsymbol{A} = \boldsymbol{\alpha}$. The probability $p_{f|A}()$ in Eq. (10) is called the probability of failure for a particular direction $\boldsymbol{A} = \boldsymbol{\alpha}$ in the load space [7].

A Monte Carlo estimate of p_f is obtained simulating N outcomes $\boldsymbol{\alpha}_j$ of the unit vector $\boldsymbol{A}$ and averaging the corresponding sample values $p_{f|A}(\boldsymbol{\alpha}_j)$:

$$\hat{p}_{f2,N} = \frac{1}{N} \sum_{j=1}^{N} p_{f|A}(\alpha_j) \quad (11)$$

A computation of the sample value $p_{f|A}(\alpha_j)$ consists in an evaluation of the integral

$$p_{f|A}(\alpha) = \int_0^{\infty} \left(1 - F_S(s|\alpha)\right) f_R(s|\alpha) ds \quad (12)$$

for the simulated direction $\boldsymbol{\alpha}_j$. That is the point to estimating the plastic failure probability by the radial sampling, in the case at hand, consists in an integration over the radial directions.

5. Radial integration

The conditional plastic failure probability, $p_{f|A}(\alpha)$ given by Eq(12) can be evaluated carrying out the one-dimensional integration over the radial direction α provided that the functions $F_S(.|.)$ and $f_R(.|.)$ can be evaluated for each s and α. The conditional CDF $F_S(.|.)$ may be expressed in terms of the joint density $f_L()$ with the use of the mapping $l = \Phi_n(s,\alpha)$ from the (s,α)-space to the l-space (see e.g.[16, Sec.7.19]). Firstly, the function $F_S(.|.)$ may be given by

$$F_S(s|\alpha) = \int_0^s f_S(z|\alpha)dz$$

$$= (f_A(\alpha))^{-1}\int_0^s f_{S,A}(z,\alpha)dz$$

$$= \left(\int_0^\infty f_{S,A}(z,\alpha)dz\right)^{-1}\int_0^s f_{S,A}(z,\alpha)dz \qquad (13)$$

where $f_S(.|.)$ is the conditional PDF of S; $f_A()$ is the marginal PDF of A; and $f_{S,A}()$ is the joint PDF of S and A. Secondly, the latter PDF may be represented as

$$f_{S,A}(s,\alpha) = |det\Phi'(s,\alpha)|\, f_L(\Phi_n(s,\alpha))$$

$$= \left|det\frac{\partial l}{\partial(s,\alpha)}\right| f_L(\Phi_n(s,\alpha)) \qquad (14)$$

where $det\mathbf{B}$ denotes the determinant of $\mathbf{B}$. Finally, the function $F_S(.|.)$ takes the form

$$F_S(s|\alpha) = \left(\int_0^\infty |det\Phi'(z,\alpha)|\, f_L(\Phi_n(z,\alpha))dz\right)^{-1}$$

$$\times\int_0^s |det\Phi'(z,\alpha)|\, f_L(\Phi_n(z,\alpha))dz \qquad (15)$$

The integrals in Eq (15) are one-dimensional and so they can be evaluated by a numerical technique.

A problematical term of the integrand in Eq (12) is the conditional probability density function $f_R(.|.)$ of the resistance $R(\boldsymbol{M}_0,\alpha)$. As may be seen from the LP problem (1), the distribution of $R(\boldsymbol{M}_0,\alpha)$ depends for given α on the joint distribution of the limiting moments.

To obtain an analytical expression of the PDF $f_R(.|.)$ and hence the corresponding CDF $F_R(.|.)$ is a trivial task when the plastic moment capacities in all critical sections are modelled only by a single random variable, i.e., M_0. The resistance $R(M_0,\alpha)$ in this situation is a linear function of M_0, and both of them have the same type of probability distribution. The expression (12) in this case may be rewritten in the form

$$p_{f|A}(\alpha) = \int_0^\infty f_S(s|\alpha)\, F_R(s|\alpha)ds \qquad (16)$$

that is simpler from the computational point of view.

At the other extreme is the case when the plastic moment capacities M_{0i} are stochastically independent. The values of $F_R(.|.)$ can in this case be evaluated by integrating the marginal densities $f_{M0i}(\ )$ [17].

If the dimensionality of the load-space, m, is not large, the values of α may be discretised and moments of the distribution of $R(\boldsymbol{M}_0,\alpha)$ approximated by the method given in Refs [9,11]. The method has a limitation that there is only one mode of plastic collapse considered for each discretised direction α.

In the general case, the radial integration may be performed via the simple Monte Carlo simulation. One possibility is that observations for the estimation of $p_{f|A}(\alpha_i)$ are sampled from the distribution $f_R(.|.)$ by solving the LP problem (1). The Monte Carlo estimate takes in this case the form

$$\hat{p}_{f3,NK} = N^{-1}\sum_{i=1}^{N}\hat{p}_{f|A,K}(\alpha_i)$$

$$= N^{-1}\sum_{i=1}^{N}\frac{1}{K}\, K^{-1}\sum_{j=1}^{K}\left(1 - F_S\left(r(\boldsymbol{m}_{0j},\alpha_i)\mid \alpha_i\right)\right) \quad (17)$$

where $\hat{p}_{f|A,K}(\alpha_i)$ is the estimate of $p_{f|A}(\alpha_i)$.

The properties of the radial sampling estimator

$$\hat{P}_{f3,NK} = N^{-1}\sum_{i=1}^{N}\frac{1}{K}\, K^{-1}\sum_{j=1}^{K}\left(1 - F_S\left(r(\boldsymbol{M}_{0j},\boldsymbol{A}_i)\,|\boldsymbol{A}_i\right)\right)$$

may be discussed comparing them with ones of the simple Monte Carlo estimator

$$\hat{P}_{f1,N} = N^{-1}\sum_{j=1}^{N}\mathbf{1}(\boldsymbol{M}_{0j},\ \boldsymbol{L}_j)$$

where $\boldsymbol{M}_{0j}$, $\boldsymbol{L}_j$ and $\boldsymbol{A}_i$ are random vectors with PDF's that are identical to the PDF's of $\boldsymbol{M}_0$, $\boldsymbol{L}$ and $\boldsymbol{A}$, respectively.

It is well known that the estimation of the plastic failure probability p_f using $\hat{P}_{f1,N}$ is performed by generating the sample $\mathbf{1}(\boldsymbol{m}_{01},l_1)$, ... , $\mathbf{1}(\boldsymbol{m}_{0N},l_N)$ which usually consists of zeros with only few ones. If the unknown probability p_f is small enough, we will often obtain zero values of the estimate $\hat{p}_{f1,N}$ for N less than $1/p_f$. This is not the case if the estimator $\hat{P}_{f3,NK}$ is used to assess p_f. Each term in the double sum of Eq (17) contributes to the result $\hat{p}_{f3,NK}$ with a value lying between zero and one. Hence, a non-zero estimate $\hat{p}_{f3,NK}$ will be obtained even for small size of the sample

$$\left(1 - F_S\left(r\left(\boldsymbol{m}_{0j}, \alpha_i\right) \mid \alpha_i\right)\right), \ldots$$

$$\ldots, \left(1 - F_S\left(r\left(\boldsymbol{m}_{0K}, \alpha_N\right) \mid \alpha_N\right)\right),$$

no matter how small p_f is.

The generation of the sample elements $\mathbf{1}\left(\boldsymbol{m}_{0j}, \boldsymbol{l}_j\right)$ and $\left(1 - F_S\left(r\left(\boldsymbol{m}_{0j}, \alpha_i\right) \mid \alpha_i\right)\right)$ requires to solve once the LP problem (1). Thus the estimator $\hat{P}_{f3,NK}$ is more efficient than $\hat{P}_{f1,N}$, the number of solvings of the LP problem being the same. Futhermore, numerical experiments show that the sequence

$$k^{-1}\sum_{j=1}^{k}\left(1 - F_S\left(r\left(\boldsymbol{m}_{0j}, \alpha_i\right) \mid \alpha_i\right)\right), \quad k = 1, 2, \ldots$$

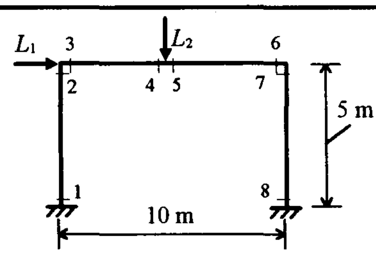
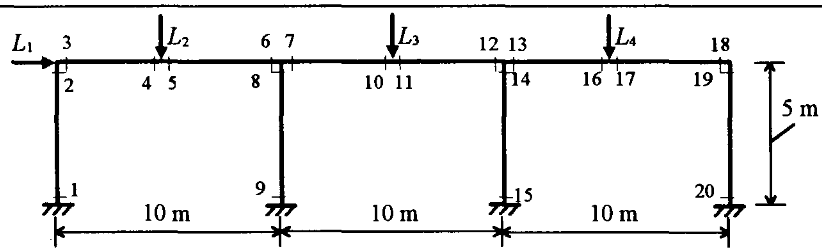
converges to $p_{f|A}(\alpha_i)$ relatively fast.

One might expect that the variance of the estimator $\hat{P}_{f3,NK}$ is high even though the sampling PDF $f_A(\cdot)$ is nonuniform. The failure probabilities $p_{f|A}(\alpha_i)$ can vary significantly for different loading proportions α_i. A variance reduction technique, say, importance sampling should be applied here together with the radial sampling procedure to lower the variance of $\hat{P}_{f3,NK}$. This task, however, is outside the purposes of this paper.

6. Examples

Consider the frames in Table 1. These structures have been previously used for comparison studies reported in Ref [9]. The random plastic moment capacities and loads are assumed to be normally or lognormally distributed with the characteristics given in Table 2. The plastic moment capacities are considered as either fully correlated or uncorrelated, that is $\rho[M_{0i}, M_{0j}] = 1$ or $\rho[M_{0i}, M_{0j}] = 0$. Four values 0.3/0.05, 0.3/0.1, 0.2/0.1, and 0.1/0.1 of the ratio v_L/v_M of coefficients of variation of loads to plastic moment capacities were used.

Table 1. Frames considered

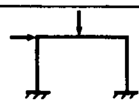
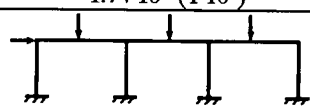
Number of frame	Discretisation scheme and identical sections[a]
1 (Frame 1)	 $M_{04} = M_{05}$
2 (Frame 2)	 $M_{04} = M_{05}$, $M_{0,10} = M_{0,11}$, $M_{0,16} = M_{0,17}$

[a] plastic moment capacities in the identical sections are modelled by the same random variable

Table 2. Characteristics of plastic moment capacities and loads

Plastic moment capacities	
Frame 1	Frame 2
$E[M_{0i}]$ = 135 kNm, $i = 1, \dots, 8$; $v_{Mi} = v_M = 0.05, 0.1$, $i = 1, \dots, 8$; $\rho[M_{0i}, M_{0j}] = 1$, $\rho[M_{0i}, M_{0j}] = 0$, $i \neq j$ and $i,j = 1, \dots, 8$	$E[M_{0i}]$ = 100 kNm, $i = 1, \dots, 20$; $v_{Mi} = v_M = 0.05, 0.1$, $i = 1, \dots, 20$; $\rho[M_{0i}, M_{0j}] = 1$, $\rho[M_{0i}, M_{0j}] = 0$, $i \neq j$ and $i,j = 1, \dots, 20$
Loads	
Frame 1	Frame 2
$E[L_1]$ = 50 kN, $E[L_2]$ = 40 kN; $v_{L1} = v_{L2} = v_L = 0.1, 0.3$; $\rho[L_1, L_2] = 0$; $\rho[M_i, L_j] = 0$ for each i and j	$E[L_1]$=50 kN, $E[L_i]$ = 40 kN, $i = 2, \dots, 4$; $v_{Li} = v_L = 0.1, 0.3$, $i = 1, \dots, 4$; $\rho[L_i, L_j] = 0$, $i \neq j$ and $i,j = 1, \dots, 20$; $\rho[M_i, L_j] = 0$ for each i and j
Note: $E[.]$, v, and $\rho[.,.]$ denotes mean, coefficient of variation, and correlation coefficient, respectively	

Table 3. Estimates of failure probabilities computed with $\rho[M_{0i}, M_{0j}] = 1$ for normally-distributed loads and moment capacities

Ratio of C.O.V. of load to resistance v_L/v_M	Monte Carlo estimates $\hat{p}_{f1,N}(N)$	Radial sampling estimates $\hat{p}_{f2,N}(N)$		Method of discretisation of the load-space[a]: Load space-plastic	Method of discretisation of the load-space[a]: Load space-nonlinear	Monte Carlo simulation[a]
						
$0.3/0.05$	$2.50\cdot10^{-4}(1\cdot10^{5})$ $2.60\cdot10^{-4}(1\cdot10^{5})$ $3.00\cdot10^{-4}(1\cdot10^{5})$	$3.36\cdot10^{-4}(1\cdot10^{2})$ $3.37\cdot10^{-4}(1\cdot10^{2})$ $3.77\cdot10^{-4}(1\cdot10^{2})$		$3.88\cdot10^{-4}$	$7.06\cdot10^{-4}$	$5.5\cdot10^{-4}$
$0.3/0.1$	$2.18\cdot10^{-3}(1\cdot10^{5})$ $2.32\cdot10^{-3}(1\cdot10^{5})$ $2.32\cdot10^{-3}(1\cdot10^{5})$	$2.33\cdot10^{-3}(1\cdot10^{2})$ $2.37\cdot10^{-3}(1\cdot10^{2})$ $2.46\cdot10^{-3}(1\cdot10^{2})$		$2.61\cdot10^{-3}$	$3.79\cdot10^{-3}$	$3.60\cdot10^{-4}$
$0.2/0.1$	$1.80\cdot10^{-4}(1\cdot10^{5})$ $1.90\cdot10^{-4}(1\cdot10^{5})$ $2.40\cdot10^{-4}(1\cdot10^{5})$	$2.41\cdot10^{-4}(1\cdot10^{2})$ $2.42\cdot10^{-4}(1\cdot10^{2})$ $2.48\cdot10^{-4}(1\cdot10^{2})$		—	—	—
$0.1/0.1$	$0.55\cdot10^{-5}(1\cdot10^{7})$ $0.57\cdot10^{-5}(1\cdot10^{7})$ $0.62\cdot10^{-5}(1\cdot10^{7})$	$1.73\cdot10^{-5}(1\cdot10^{2})$ $1.73\cdot10^{-5}(1\cdot10^{2})$ $1.74\cdot10^{-5}(1\cdot10^{2})$		$2.54\cdot10^{-5}$	$3.70\cdot10^{-5}$	—
						
$0.3/0.05$	$1.49\cdot10^{-3}(1\cdot10^{5})$ $1.81\cdot10^{-3}(1\cdot10^{5})$ $1.88\cdot10^{-3}(1\cdot10^{5})$	$1.79\cdot10^{-3}(1\cdot10^{2})$ $2.57\cdot10^{-3}(1\cdot10^{2})$ $3.13\cdot10^{-3}(1\cdot10^{2})$	$2.04\cdot10^{-3}(2\cdot10^{2})$ $2.16\cdot10^{-3}(2\cdot10^{2})$ $2.70\cdot10^{-3}(2\cdot10^{2})$	—	$3.23\cdot10^{-3}$	$2.8\cdot10^{-3}$
$0.3/0.1$	$6.57\cdot10^{-3}(1\cdot10^{5})$ $7.13\cdot10^{-3}(1\cdot10^{5})$ $7.53\cdot10^{-3}(1\cdot10^{5})$	$7.10\cdot10^{-3}(1\cdot10^{2})$ $8.26\cdot10^{-3}(1\cdot10^{2})$ $9.61\cdot10^{-3}(1\cdot10^{2})$	$7.28\cdot10^{-3}(2\cdot10^{2})$ $7.41\cdot10^{-3}(2\cdot10^{2})$ $8.74\cdot10^{-3}(2\cdot10^{2})$	—	$8.92\cdot10^{-3}$	$9.0\cdot10^{-3}$
$0.2/0.1$	$4.10\cdot10^{-4}(1\cdot10^{5})$ $4.20\cdot10^{-4}(1\cdot10^{5})$ $4.90\cdot10^{-4}(1\cdot10^{5})$	$5.13\cdot10^{-4}(1\cdot10^{2})$ $5.96\cdot10^{-4}(1\cdot10^{2})$ $6.26\cdot10^{-4}(1\cdot10^{2})$	$4.70\cdot10^{-4}(2\cdot10^{2})$ $5.41\cdot10^{-4}(2\cdot10^{2})$ $6.61\cdot10^{-4}(2\cdot10^{2})$	—	—	—
$0.1/0.1$	$0\ (1\cdot10^{6})$ $0.2\cdot10^{-5}(1\cdot10^{6})$ $0.4\cdot10^{-5}(1\cdot10^{6})$ $0.5\cdot10^{-5}(1\cdot10^{6})$	$0.97\cdot10^{-5}(1\cdot10^{2})$ $1.00\cdot10^{-5}(1\cdot10^{2})$ $1.12\cdot10^{-5}(1\cdot10^{2})$	$0.94\cdot10^{-5}(2\cdot10^{2})$ $0.98\cdot10^{-5}(2\cdot10^{2})$ $1.03\cdot10^{-5}(2\cdot10^{2})$	—	$7.85\cdot10^{-5}$	—

[a] Probabilities are taken from Ref [9].
— = data not available.

The load-space for the Frame 1 and Frame 2 is two- and four-dimensional, respectively. The mapping $l = \Phi_2(s,\alpha)$ from the (s,α)-space to the l-space has in the two-dimensional case the form

$$l_1 = s \cdot cos\phi = s \cdot \alpha_1$$
$$l_2 = s \cdot sin\phi = s \cdot \alpha_2$$

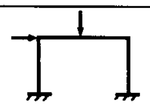
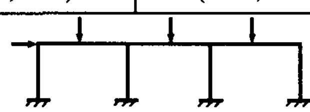
with

$$det\Phi'(s,\alpha) = s$$

where s and ϕ are polar coordinates in the two-dimensional load-space. The mapping $\Phi_4(s,\alpha)$ is expressed by equalities

$$l_1 = s \cdot cos\phi_1 \cdot sin\phi_2 \cdot sin\phi_3 = s \cdot \alpha_1$$
$$l_2 = s \cdot sin\phi_1 \cdot sin\phi_2 \cdot sin\phi_3 = s \cdot \alpha_2$$
$$l_3 = s \cdot cos\phi_2 \cdot sin\phi_3 = s \cdot \alpha_3$$
$$l_4 = s \cdot cos\phi_3 = s \cdot \alpha_4$$

Table 4. Estimates of failure probabilities computed with $\rho[M_{0i}, M_{0j}] = 0$ for normally-distributed loads and moment capacities

Ratio of C.O.V. of load to resistance v_L/v_M	Monte Carlo estimates $\hat{p}_{f1,N}(N)$	Radial sampling estimates $\hat{p}_{f3,NK}(N,K)$		Method of discretisation of the load-space[a]: Load space-plastic	Method of discretisation of the load-space[a]: Load space-nonlinear	Monte Carlo simulation[a]
						
0.3/0.05	$1.20\cdot10^{-4}$ $(1\cdot10^5)$ $1.50\cdot10^{-4}$ $(1\cdot10^5)$ $1.70\cdot10^{-4}$ $(1\cdot10^5)$	$1.77\cdot10^{-4}$ $(1\cdot10^2,1\cdot10^2)$ $1.85\cdot10^{-4}$ $(1\cdot10^2,1\cdot10^2)$ $1.93\cdot10^{-4}$ $(1\cdot10^2,1\cdot10^2)$		$2.42\cdot10^{-4}$	$4.82\cdot10^{-4}$	$1.8\cdot10^{-4}$
0.3/0.1	$4.00\cdot10^{-4}$ $(1\cdot10^5)$ $5.10\cdot10^{-4}$ $(1\cdot10^5)$ $5.60\cdot10^{-4}$ $(1\cdot10^5)$	$4.23\cdot10^{-4}$ $(1\cdot10^2,1\cdot10^2)$ $4.25\cdot10^{-4}$ $(1\cdot10^2,1\cdot10^2)$ $4.31\cdot10^{-4}$ $(1\cdot10^2,1\cdot10^2)$		$8.80\cdot10^{-4}$	$15.6\cdot10^{-4}$	$5.1\cdot10^{-4}$
0.2/0.1	$1.00\cdot10^{-6}$ $(1\cdot10^7)$ $1.90\cdot10^{-6}$ $(1\cdot10^7)$ $2.00\cdot10^{-6}$ $(1\cdot10^7)$	$1.42\cdot10^{-6}$ $(1\cdot10^2,1\cdot10^2)$ $1.55\cdot10^{-6}$ $(1\cdot10^2,1\cdot10^2)$ $2.11\cdot10^{-6}$ $(1\cdot10^2,1\cdot10^2)$		—	—	—
0.1/0.1	—	$0.55\cdot10^{-14}$ $(1\cdot10^2,1\cdot10^2)$ $6.11\cdot10^{-13}$ $(1\cdot10^2,1\cdot10^2)$ $7.18\cdot10^{-13}$ $(1\cdot10^2,1\cdot10^2)$	$0.11\cdot10^{-13}$ $(1\cdot10^2,2\cdot10^2)$ $3.42\cdot10^{-13}$ $(1\cdot10^2,2\cdot10^2)$ $3.74\cdot10^{-13}$ $(1\cdot10^2,2\cdot10^2)$	$1.50\cdot10^{-7}$	$6.40\cdot10^{-7}$	—
						
0.3/0.05	$1.06\cdot10^{-3}$ $(5\cdot10^4)$ $1.14\cdot10^{-3}$ $(5\cdot10^4)$ $1.20\cdot10^{-3}$ $(5\cdot10^4)$	$0.90\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $2.02\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $2.22\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$	$1.12\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$ $1.25\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$ $1.59\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$	—	$2.62\cdot10^{-3}$	$1.0\cdot10^{-3}$
0.3/0.1	$2.92\cdot10^{-3}$ $(5\cdot10^4)$ $2.98\cdot10^{-3}$ $(5\cdot10^4)$ $3.06\cdot10^{-3}$ $(5\cdot10^4)$	$3.31\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $4.09\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $5.57\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$	$2.25\cdot10^{-3}$ $(2\cdot10^2,1\cdot10^2)$ $3.17\cdot10^{-3}$ $(2\cdot10^2,1\cdot10^2)$ $3.90\cdot10^{-3}$ $(2\cdot10^2,1\cdot10^2)$	—	$5.27\cdot10^{-3}$	$3.0\cdot10^{-3}$
0.2/0.1	$1.00\cdot10^{-5}$ $(1\cdot10^5)$ $4.00\cdot10^{-5}$ $(1\cdot10^5)$ $6.00\cdot10^{-5}$ $(1\cdot10^5)$	$7.00\cdot10^{-5}$ $(1\cdot10^2,1\cdot10^2)$ $10.3\cdot10^{-5}$ $(1\cdot10^2,1\cdot10^2)$ $22.2\cdot10^{-5}$ $(1\cdot10^2,1\cdot10^2)$	$3.21\cdot10^{-5}$ $(2\cdot10^2,1\cdot10^2)$ $5.01\cdot10^{-5}$ $(2\cdot10^2,1\cdot10^2)$ $5.31\cdot10^{-5}$ $(2\cdot10^2,1\cdot10^2)$	—	—	—
0.1/0.1	—	$4.87\cdot10^{-5}$ $(2\cdot10^2,2\cdot10^2)$ $4.95\cdot10^{-5}$ $(2\cdot10^2,2\cdot10^2)$ $5.00\cdot10^{-5}$ $(2\cdot10^2,2\cdot10^2)$		—	$2.66\cdot10^{-6}$	—

[a] Probabilities are taken from Ref [9].
— = data not available.

with

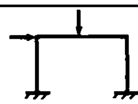
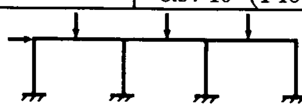
with

$$det\Phi'(s,\alpha) = s^3 \cdot sin\phi_2 \cdot (sin\phi_3)^2$$

where s and $\phi_1, \dots, \phi_3$ are polar coordinates in the four-dimensional load-space.

The estimates $\hat{p}_{f1,N}$, $\hat{p}_{f2,N}$, and $\hat{p}_{f3,NK}$ defined by Eqs (7), (11), and (17), respectively, were computed for both distribution types of loads and plastic moment capacities. The simple Monte Carlo estimate $\hat{p}_{f1,N}$ was computed in both cases $\rho(M_{0i}, M_{0j}) = 1$ and $\rho(M_{0i}, M_{0j}) = 0$. The radial sampling estimate $\hat{p}_{f2,N}$ was computed in the case $\rho(M_{0i}, M_{0j}) = 1$ using an evaluation of the integral in the Eq (16) by means of 32-point Gauss quadrature. This formula was also used for the evaluation of the evaluation of integrals in the expression of the

Table 5. Estimates of failure probabilities computed with $\rho[M_{0i}, M_{0j}] = 1$ for lognormally-distributed loads and moment capacities

Ratio of C.O.V. of load to resistance v_L/v_M	Monte Carlo estimates $\hat{p}_{f1,N}(N)$	Radial sampling estimates $\hat{p}_{f2,N}(N)$		Method of discretisation of the load-space[a]: Load space-plastic	Method of discretisation of the load-space[a]: Load space-nonlinear	Monte Carlo simulation[a]
						
0.3/0.05	$3.91\cdot10^{-3}(1\cdot10^5)$ $3.99\cdot10^{-3}(1\cdot10^5)$ $4.27\cdot10^{-3}(1\cdot10^5)$	$4.02\cdot10^{-3}(1\cdot10^2)$ $4.42\cdot10^{-3}(1\cdot10^2)$ $5.75\cdot10^{-3}(1\cdot10^2)$	$4.28\cdot10^{-3}(1\cdot10^3)$ $5.00\cdot10^{-3}(1\cdot10^3)$ $5.21\cdot10^{-3}(1\cdot10^3)$	$4.8\cdot10^{-3}$	$6.6\cdot10^{-3}$	$7.0\cdot10^{-3}$
0.3/0.1	$6.64\cdot10^{-3}(1\cdot10^5)$ $6.67\cdot10^{-3}(1\cdot10^5)$ $6.91\cdot10^{-3}(1\cdot10^5)$	$6.71\cdot10^{-3}(1\cdot10^2)$ $7.07\cdot10^{-3}(1\cdot10^2)$ $8.52\cdot10^{-3}(1\cdot10^2)$	$6.89\cdot10^{-3}(1\cdot10^3)$ $7.79\cdot10^{-3}(1\cdot10^3)$ $8.05\cdot10^{-3}(1\cdot10^3)$	$7.5\cdot10^{-3}$	$10.0\cdot10^{-3}$	$8.8\cdot10^{-4}$
0.2/0.1	$2.70\cdot10^{-4}(1\cdot10^5)$ $3.00\cdot10^{-4}(1\cdot10^5)$ $3.10\cdot10^{-4}(1\cdot10^5)$	$3.36\cdot10^{-4}(1\cdot10^2)$ $3.53\cdot10^{-4}(1\cdot10^2)$ $5.21\cdot10^{-4}(1\cdot10^2)$	$3.84\cdot10^{-4}(1\cdot10^3)$ $4.43\cdot10^{-3}(1\cdot10^3)$ $4.53\cdot10^{-3}(1\cdot10^3)$	—	—	—
0.1/0.1	—	$8.02\cdot10^{-7}(1\cdot10^2)$ $8.19\cdot10^{-7}(1\cdot10^2)$ $8.48\cdot10^{-7}(1\cdot10^2)$	$8.28\cdot10^{-7}(1\cdot10^3)$ $8.54\cdot10^{-7}(1\cdot10^3)$ $8.57\cdot10^{-7}(1\cdot10^3)$	$9.0\cdot10^{-7}$	$19.0\cdot10^{-7}$	—
						
0.3/0.05	$1.75\cdot10^{-2}(5\cdot10^4)$ $1.90\cdot10^{-2}(5\cdot10^4)$ $1.90\cdot10^{-2}(5\cdot10^4)$	$1.68\cdot10^{-2}(1\cdot10^2)$ $1.99\cdot10^{-2}(1\cdot10^2)$ $2.32\cdot10^{-2}(1\cdot10^2)$	$1.53\cdot10^{-2}(2\cdot10^2)$ $2.03\cdot10^{-2}(2\cdot10^2)$ $2.51\cdot10^{-2}(2\cdot10^2)$	—	$1.3\ 10^{-2}$	$2.6\cdot10^{-2}$
0.3/0.1	$2.44\cdot10^{-2}(5\cdot10^4)$ $2.54\cdot10^{-2}(5\cdot10^4)$ $2.57\cdot10^{-2}(5\cdot10^4)$	$2.36\cdot10^{-2}(1\cdot10^2)$ $2.72\cdot10^{-2}(1\cdot10^2)$ $2.98\cdot10^{-2}(1\cdot10^2)$	$2.12\cdot10^{-2}(2\cdot10^2)$ $2.72\cdot10^{-2}(2\cdot10^2)$ $3.25\cdot10^{-2}(2\cdot10^2)$	—	$1.8\cdot10^{-2}$	$4.0\cdot10^{-2}$
0.2/0.1	$1.36\cdot10^{-3}(5\cdot10^4)$ $1.68\cdot10^{-3}(5\cdot10^4)$ $1.68\cdot10^{-3}(5\cdot10^4)$	$1.76\cdot10^{-3}(1\cdot10^2)$ $1.92\cdot10^{-3}(1\cdot10^2)$ $2.48\cdot10^{-3}(1\cdot10^2)$	$1.43\cdot10^{-3}(2\cdot10^2)$ $1.82\cdot10^{-3}(2\cdot10^2)$ $2.31\cdot10^{-3}(2\cdot10^2)$	—	—	—
0.1/0.1	$0(1\cdot10^6)$	$5.60\cdot10^{-6}(1\cdot10^2)$ $5.66\cdot10^{-6}(1\cdot10^2)$ $7.31\cdot10^{-6}(1\cdot10^2)$	$6.04\cdot10^{-6}(2\cdot10^2)$ $6.58\cdot10^{-6}(2\cdot10^2)$ $7.56\cdot10^{-6}(2\cdot10^2)$	—	$1.4\cdot10^{-5}$	—

[a] Probabilities are taken from Ref [9].
— = data not available.

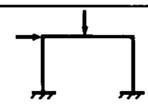
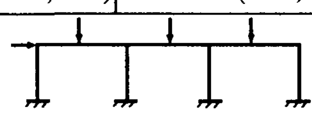
conditional CDF $F_S(.|.)$ (Eq(15)). The radial sampling estimate $\hat{p}_{f3,N}$ was computed in the case $\rho(M_{0i}, M_{0j}) = 0$. The observations α_j of the random vector A were sampled from the distribution $f_A(\alpha)$ by the formula

$$\alpha_i = l_i / \|l_i\|$$

where l_i is the ith observation of the load vector.

Three values of the estimates $\hat{p}_{f1,N}$, $\hat{p}_{f2,N}$, and $\hat{p}_{f3,NK}$ were computed for each combination of distribution type, correlation coefficient $\rho(M_{0i}, M_{0j})$, ratio v_L/v_M, and sample sizes N and $N{\cdot}K$. The computed values were sorted in ascending order and are summarized in Tables 3 to 6. The estimates cited from Ref [9] are also shown.

Table 6. Estimates of failure probabilities computed with $\rho[M_{0i}, M_{0j}] = 0$ for lognormally-distributed loads and moment capacities

Ratio of C.O.V. of load to resistance v_L/v_M	Monte Carlo estimates $\hat{p}_{f1,N}(N)$	Radial sampling estimates $\hat{p}_{f3,NK}(N,K)$		Method of discretisation of the load-space[a]: Load space-plastic	Method of discretisation of the load-space[a]: Load space-nonlinear	Monte Carlo simulation[a]
						
0.3/0.05	$4.03\cdot10^{-3}$ $(1\cdot10^5)$ $4.11\cdot10^{-3}$ $(1\cdot10^5)$ $4.20\cdot10^{-3}$ $(1\cdot10^5)$	$2.77\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $4.12\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $8.47\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$	$4.23\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$ $4.28\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$ $4.51\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$	$4.40\cdot10^{-3}$	$6.10\cdot10^{-3}$	$6.7\cdot10^{-3}$
0.3/0.1	$5.49\cdot10^{-3}$ $(1\cdot10^5)$ $5.49\cdot10^{-3}$ $(1\cdot10^5)$ $5.87\cdot10^{-3}$ $(1\cdot10^5)$	$3.38\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $4.82\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $9.20\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$	$4.98\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$ $5.00\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$ $5.24\cdot10^{-3}$ $(1\cdot10^2,2\cdot10^2)$	$5.70\cdot10^{-3}$	$8.00\cdot10^{-3}$	$7.0\cdot10^{-3}$
0.2/0.1	$2.00\cdot10^{-5}$ $(1\cdot10^5)$ $5.00\cdot10^{-5}$ $(1\cdot10^5)$ $9.00\cdot10^{-5}$ $(1\cdot10^5)$	$4.49\cdot10^{-5}$ $(1\cdot10^2,1\cdot10^2)$ $6.66\cdot10^{-5}$ $(1\cdot10^2,1\cdot10^2)$ $25.8\cdot10^{-5}$ $(1\cdot10^2,1\cdot10^2)$	$6.56\cdot10^{-5}$ $(1\cdot10^2,2\cdot10^2)$ $6.95\cdot10^{-5}$ $(1\cdot10^2,2\cdot10^2)$ $9.47\cdot10^{-5}$ $(1\cdot10^2,2\cdot10^2)$	—	—	—
0.1/0.1	—	$0.34\cdot10^{-11}$ $(1\cdot10^2,1\cdot10^2)$ $1.54\cdot10^{-11}$ $(1\cdot10^2,1\cdot10^2)$ $5.65\cdot10^{-11}$ $(1\cdot10^2,1\cdot10^2)$	$0.57\cdot10^{-11}$ $(1\cdot10^2,2\cdot10^2)$ $0.79\cdot10^{-11}$ $(1\cdot10^2,2\cdot10^2)$ $1.05\cdot10^{-11}$ $(1\cdot10^2,2\cdot10^2)$	$2.40\cdot10^{-7}$	$9.00\cdot10^{-7}$	—
						
0.3/0.05	$1.67\cdot10^{-2}$ $(5\cdot10^4)$ $1.82\cdot10^{-2}$ $(5\cdot10^4)$ $1.85\cdot10^{-2}$ $(5\cdot10^4)$	$1.31\cdot10^{-2}$ $(1\cdot10^2,1\cdot10^2)$ $1.63\cdot10^{-2}$ $(1\cdot10^2,1\cdot10^2)$ $2.12\cdot10^{-2}$ $(1\cdot10^2,1\cdot10^2)$	$1.50\cdot10^{-2}$ $(2\cdot10^2,1\cdot10^2)$ $1.60\cdot10^{-2}$ $(2\cdot10^2,1\cdot10^2)$ $1.91\cdot10^{-2}$ $(2\cdot10^2,1\cdot10^2)$	—	$1.2\cdot10^{-2}$	$2.5\cdot10^{-2}$
0.3/0.1	$2.11\cdot10^{-2}$ $(5\cdot10^4)$ $2.20\cdot10^{-2}$ $(5\cdot10^4)$ $2.22\cdot10^{-2}$ $(5\cdot10^4)$	$2.47\cdot10^{-2}$ $(1\cdot10^2,1\cdot10^2)$ $2.92\cdot10^{-2}$ $(1\cdot10^2,1\cdot10^2)$ $3.43\cdot10^{-2}$ $(1\cdot10^2,1\cdot10^2)$	$1.93\cdot10^{-2}$ $(2\cdot10^2,1\cdot10^2)$ $2.23\cdot10^{-2}$ $(2\cdot10^2,1\cdot10^2)$ $2.82\cdot10^{-2}$ $(2\cdot10^2,1\cdot10^2)$	—	$1.5\cdot10^{-2}$	$3.0\cdot10^{-2}$
0.2/0.1	$0.62\cdot10^{-3}$ $(5\cdot10^4)$ $0.68\cdot10^{-3}$ $(5\cdot10^4)$ $0.76\cdot10^{-3}$ $(5\cdot10^4)$	$0.41\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $1.40\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$ $2.11\cdot10^{-3}$ $(1\cdot10^2,1\cdot10^2)$	$1.08\cdot10^{-3}$ $(2\cdot10^2,1\cdot10^2)$ $1.16\cdot10^{-3}$ $(2\cdot10^2,1\cdot10^2)$ $1.35\cdot10^{-2}$ $(2\cdot10^2,1\cdot10^2)$	—	—	—
0.1/0.1	—	$0.48\cdot10^{-12}$ $(1\cdot10^2,1\cdot10^2)$ $11.0\cdot10^{-12}$ $(1\cdot10^2,1\cdot10^2)$ $60.7\cdot10^{-12}$ $(1\cdot10^2,1\cdot10^2)$	$6.74\cdot10^{-12}$ $(2\cdot10^2,1\cdot10^2)$ $6.77\cdot10^{-12}$ $(2\cdot10^2,1\cdot10^2)$ $30.3\cdot10^{-12}$ $(2\cdot10^2,1\cdot10^2)$	—	$5.1\cdot10^{-6}$	—

[a] Probabilities are taken from Ref [9].
— = data not available.

The results show an agreement between the estimates $\hat{p}_{f1,N}$ and $\hat{p}_{f3,NK}$. The values $\hat{p}_{f3,NK}$ computed for the ratio v_L/v_M equal to 0.1/0.1 can be seen only as hypothetical because it was difficult to check them by simple Monte Carlo simulation. A somewhat greater difference between values of $\hat{p}_{f3,NK}$ and estimates cited from Ref [9] may be explained by differences between the radial sampling method and estimation methods used in Ref [9].

7. Conclusions

The proposed combination of the radial sampling procedure and the limit equilibrium method is an effective technique for the estimation of the plastic failure probability of plastic frames. It allows to compute relative small failure probabilities, which are often required in structural problems. The procedure does not require any adaptive intervention in the estimating of the plastic failure probability. Additional investigations are needed to reduce the variance of the probability estimator of the radial sampling procedure.

References

1. F.Moses. Probabilistic Analysis of Structural Systems // Probabilistic Structural Mechanics Handbook, C.(Raj)Sundararajan, ed. New York etc.: Chapman and Hall, 1995, p. 166-187.
2. O.Ditlevsen, P.Bjerager. Plastic Reliability Analysis by Directional Simulation // J. Eng. Mech., 115(6), 1989, p. 1347-1362.
3. P.Bjerager. Plastic Systems Reliability by LP and FORM // Comp. and Struct., 31(2), 1989, p. 187-196.
4. T.Arnbjerg-Nielsen. Rigid-Idealplastic Model as a Reliability Analysis Tool for Ductile Structures. Thesis submitted to the Technical University of Denmark in partial fulfilment of the requirements for the PhD degree. Lingdby: Technical University of Denmark, 1991. 100 p.
5. O.Ditlevsen, P.Bjerager. Reliability of Highly Redundant Plastic Structures // J. Eng. Mech., 110(5), 1984, p. 671-683.
6. P. Bjerager. Probability Integration by Directional Simulation // J. Eng. Mech., 114(8), 1988, p. 1285-1302.
7. R.E.Melchers. Modern Computational Techniques for Reliability Estimation // Proc. of the Conf. Probabilistic Methods in Geotechnical Engineering, K.S.Li and C.R.Lo (eds.), Rotterdam: Balkema, 1993, p. 153-163.
8. T.-Y.Kam, R.B.Corotis, E.C.Rossow. Reliability of Nonlinear Framed Structures // J. Struct. Eng., 109(7), 1983, p. 1585-1601.
9. T.S.Lin, R.B.Corotis. Reliability of Ductile Systems with Random Strengths // J. Struct. Eng., 111(6), 1985, p. 1306-1325.
10. M.Soltani, R.B.Corotis. Reliability of Random Structural Systems and Load Space Reduction // J. Struct. Eng., 113(10), 1987, p. 2145-2159.
11. R.B.Corotis, A.M.Nafday. Structural Systems Reliability Using Linear Programming and Simulation // J. Struct. Eng., 115(10), 1989, p. 2435-2447.
12. M.Z.Cohn, S.K.Ghosh, S.R.Parimi. Unified Approach to Theory of Plastic Structures // J. Eng. Mech. Div. 98(5), 1972, p. 1133-1158.
13. A.Cyras. Analysis and Optimization of Elastoplastic Systems. Chichester: Ellis Horwood Ltd. Publishers, 1982. 121 p.
14. R.E.Melchers. Radial Importance Sampling for Structural Reliability // J. Eng. Mech., 116(1), 1990, p. 189-203.
15. R.E.Melchers. Load-Space Formulation for Time Dependent Structural Reliability. J. Eng. Mech., 118(5), 1992, p. 853-870.
16. W.Walter. Analysis II. Berlin etc.: Springer-Verlag., 1990. 396 p.
17. J.B.Ewbank, B.L.Floote, H.J.Kumin. A Method for the Solution of the Distribution Problem of Stochastic Linear Programming // SIAM J. Appl. Math., 26(2), 1974, p. 225-238.

Įteikta 1997 09 11

VIENAS PLASTINIŲ RĖMŲ PATIKIMUMO VERTI NIMO STOCHASTINIU MODELIAVIMU BŪDAS

E.R. Vaidogas

Santrauka

Straipsnyje nagrinėjamas plastinių rėmų patikimumo vertinimas stochastinio modeliavimo metodu, kuris vadinamas radialiniu ėmimu. Patikimumo problema yra formuluojama apkrovų erdvėje. Naudojamas faktas, kad daugeliui praktinių problemų apkrovų erdvės dimensija yra gerokai mažesnė už bazinių kintamųjų erdvės dimensiją. Plastinės avarijos tikimybė vertinama naudojantis proporcingo apkrovos didinimo metodu. Atsisakoma apkrovų erdvės diskretizavimo pagal apkrovų proporcijas. Deterministiniu rėmų mechaniniu modeliu parinktas statinės formuluotės uždavinys, turintis tiesinio programavimo uždavinio formą. Plastinės avarijos vertinimui naudojama radialinio ėmimo ir tiesinio programavimo uždavinio kombinacija. Jungiamoji grandis tarp radialinio ėmimo ir mechaninio modelio yra apkrovos krypčių ir pasiskirstymo vektorius, kuris išreiškiamas kaip krypties kosinusų vektorius.

Išspręsti dviejų rėmų pavyzdžiai. Gauti rezultatai lyginami su kitų autorių rezultatais.

Egidijus Rytas VAIDOGAS. Senior Assistant, Doctor. Department of Reinforced Concrete Structures. Vilnius Gediminas Technical University, 11 Saulėtekio Ave, 2040 Vilnius, Lithuania.

PhD degree in 1994 (building structures) at Vilnius Technical University. Since 1995 senior assistant at the Department of Reinforced Concrete Structures. In 1993, 1996-97 research visits to Bauhaus University Weimar (Germany). Research interests: reliability-based structural analysis and design.