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SHORT-TERM DEFORMATIONAL ANALYSIS OF PRESTRESSED CONCRETE BEAMS USING FLEXURAL CONSTITUTIVE MODEL

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Abstract. In this paper, an attempt has been made to extend application of the recently proposed *Flexural constitutive model* to short-term deformational analysis of flexural prestressed concrete members. The relationship of tensile concrete is based on smeared crack approach and accumulates cracking and the tension stiffening effects. The Flexural constitutive model was applied in a simple engineering technique based on principles of strength of materials and the layered approach. To assess accuracy of the technique, deflections have been calculated for experimental prestressed concrete beams reported in the literature. Comparison has been carried out with the predictions of the well-known design code methods of different countries.

Keywords: tension stiffening, flexure, prestressed concrete beams, deflections.

1. Introduction

Design engineers for both short- and long-term analysis of reinforced (non-prestressed), prestressed and partially prestressed concrete structures can use empirical code and modern numerical methods. In last decades, the finite element method has emerged as the most powerful method of structural analysis. Numerous publications of numerical analysis has been produced by different authors from all over the world.

Literature review on methods of analysis of flexural prestressed concrete structures has indicated the following:

- various methods of design codes of different countries [1–3] are based on the standard assumptions, which ensure safe design, but neglect the tensile resistance of concrete and of bond slip, ie, they do not reveal the actual stress-strain state of cracked structures;
- numerical methods [4–7] are based on universal principles and can include all possible effects such as material non-linear, concrete cracking, creep and shrinkage, reinforcement slip, etc. However, the constitutive relationships often are too simplified and do not reflect complex multi-factor nature of the material.

From research on non-prestressed reinforced concrete beams, it is known that neglect of the tensile resistance of concrete and of bond slip yields an erroneous prediction of beam curvatures and deflections above the service load range. The actual behaviour is stiffer due to the capability of concrete to transmit stresses in tension even after cracking begins.

A new constitutive model, called the Flexural, has been developed for deformational analysis of flexural reinforced concrete members subjected to short-term loading [8, 9]. This model consists of traditional constitutive relationships for reinforcement and compressive concrete. The most important part of this model is a constitutive relationship for cracked tensile concrete which accumulates cracking, tension stiffening, reinforcement slippage and shrinkage effects. Further the Flexural constitutive model has been applied for long-term deformational analysis of flexural reinforced concrete structures [10, 11].

In this paper, an attempt has been made to extend the application of the Flexural constitutive model to short-term deformational analysis of flexural prestressed concrete members. For that purpose, deflections have been calculated for a large number of experimental prestressed concrete beams reported in literature. Comparison with the experimental deflections and the estimates of the design code methods has been performed.

2. Deflection calculation methods

In this section, some well-known deflection estimation methods for flexural prestressed concrete members are briefly described for case of short-term loading.

SNiP method [3] as an empirical method has been developed on a basis of large experimental data. Curvature of a prestressed cracked member is expressed in terms of average strains of steel ε_{spm}, and average strains of compressive concrete at the extreme fibre, ε_{cm}:

$$\kappa = \frac{\varepsilon_{spm} + \varepsilon_{cm}}{d}, \qquad (1)$$

$$\varepsilon_{spm} = \psi_s \left(\frac{M + Pe_{sp}}{z} - P \right) / E_{sp} A_{sp}, \qquad (2)$$

$$\varepsilon_{cm} = \psi_c \frac{M + Pe_{sp}}{(\zeta + \varphi_f) E_c zb\nu d}. \qquad (3)$$

From (1), (2) and (3) the curvature relationship is as follows:

$$\kappa = \frac{M + Pe_{sp}}{zd} \left[\frac{\psi_s}{E_{sp} A_{sp}} + \frac{\psi_c}{(\zeta + \varphi_f)\nu E_c bd} \right] - \frac{\psi_s P}{E_{sp} d A_{sp}}, \qquad (4)$$

where M is the external moment; P and e_{sp} are the initial prestressing force and its eccentricity; z is the distance from the compressive to the tensile resultant of the section; d is the effective depth; ψ_s is the ratio of the average steel strain, ε_{spm} and the steel strain in the cracked section, ε_{sp}; ψ_c is a similar factor defined for the extreme compressive concrete fibre; A_{sp} is the section area of the prestressed reinforcement in the tensile zone; E_{sp} and E_c are the modulus of elasticity for the prestressed steel and the concrete respectively; ζ is the compression zone depth factor; ν is a factor assessing non-elastic strains in the concrete of the compression zone and φ_f is a factor which takes into account the influence of the compressive reinforcement and the compressive flange of T-section.

In the PCI method [1], the curvature of a prestressed concrete member is determined by the classical expression $\kappa = M / EI$, where EI is the flexural stiffness. Modulus of elasticity of concrete, E_c, is constant for all loading stages, but the moment of inertia, I, varies. For the elastic stage, the deflection is calculated using the gross moment of inertia, I_g, and the deflection after cracking is calculated using the moment of inertia of the cracked section, I_{cr}. The expression for the effective moment of inertia is as follows:

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] I_{cr}, \qquad (5)$$

$$\frac{M_{cr}}{M_a} = 1 - \left(\frac{f_{tl} - f_r}{f_l} \right), \qquad (6)$$

where $f_{tl} = P / A_g + Pe / W - (M_d + M_0) / W$ and $f_l = M_d / W$ are the total stress and the stress due to live load in the member respectively; $f_r = 0{,}75\sqrt{f_c'}$ [MPa] is the modulus of rupture; P is the initial prestressing force; e is the distance from P to the neutral axis of the section; A_g is the gross section; W is the first moment of the section about the neutral axis; M_d and M_0 are the external and the mid-span self-weight moments respectively; f_c' is the compressive concrete cylinder strength.

The moment of inertia for the cracked section I_{cr} is as follows:

$$I_{cr} = nA_{ps} d_p^2 \left(1 - 1.6\sqrt{n\rho}\right), \qquad (7)$$

here $n = E_{sp} / E_c$, is the ratio of the modulus of elasticity of the prestressed steel and concrete, respectively; A_{ps} is the section area of the tensile prestressed steel; d_p is the effective depth; $\rho = A_{ps} / bd_p$ is the reinforcement ratio, where b is the section width.

In the EC2 model [2], a prestressed concrete member is divided into two regions: region I, uncracked, and region II, fully cracked. In region I, both the concrete and steel behave elastically, while in region II the steel carries all the tensile force of the member after cracking. Average curvature is expressed as

$$\kappa = (1 - \xi)\kappa_1 + \xi\kappa_2. \qquad (8)$$

Here κ_1 and κ_2 correspond to the curvatures in regions I and II respectively.

A distribution coefficient ξ indicates how close the stress-strain state is to the condition causing cracking. It takes a value of zero at the cracking moment and approaches unity as the loading increases above the cracking moment. It is given by the relation

$$\xi = 1 - \beta(\sigma_{sr} / \sigma_s), \qquad (9)$$

where β is a coefficient assessing the duration and nature of the loading (1 for short-term loads and 0,5 for sustained or cyclic loads); σ_{sr} and σ_s are the stresses in the tension steel calculated on the basis of a fully cracked section respectively under the cracking load and the load considered.

The present analysis method is based on the classical techniques of strength of materials extended to the application of the layered approach and the use of the materials diagrams of the Flexural constitutive model [8, 10]. For the short-term analysis of prestressed concrete beams, the stress-strain relationships for the compressive and tensile concrete are shown in Fig 1. The descending branch of the tensile concrete diagram shown in Fig 1, b has the following expression:

$$\sigma_t = 0{,}625 f_t' \left(1 - \frac{\varepsilon_t}{\beta} + \frac{1 + 0{,}6\beta}{\beta\varepsilon_t} \right). \qquad (10)$$

Here $\varepsilon_t = \varepsilon_t' / \varepsilon_{cr}$; $\varepsilon_t' = f_t' / E_c$; f_t' is the strength of tensile concrete; ε_{cr} and ε_t are the cracking strain and the strain of tensile concrete respectively; β is a factor describing the length of the descending branch of the constitutive relationship ($\beta = 32{,}8 - 27{,}6p + 7{,}12p^2$, where p is the reinforcement percentage).

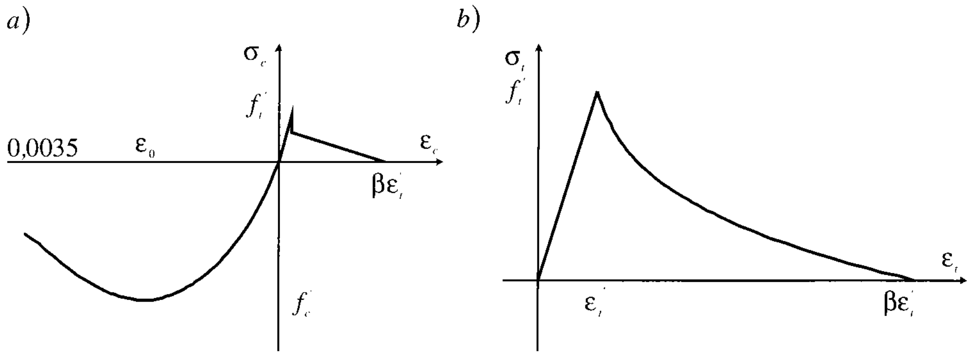

Fig 1. Stress-strain relationships for compressive and tensile concrete: a – compressive and tensile concrete; b – tensile concrete

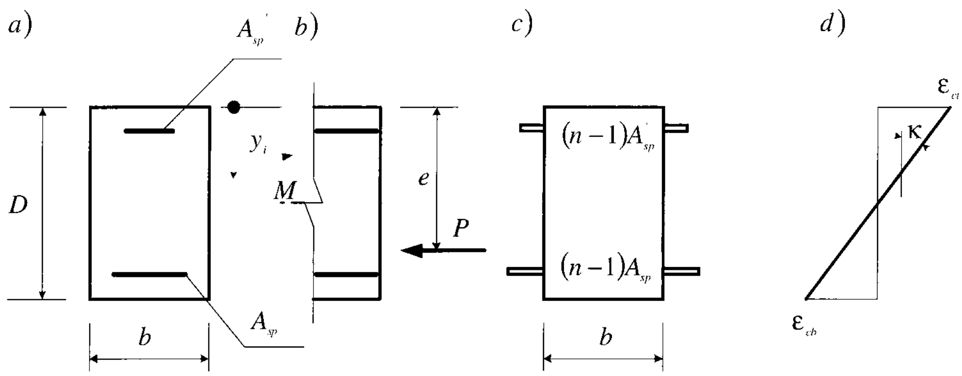

Fig 2. Prestressed concrete section: a – a doubly prestressed concrete section; b – prestressing force and external bending moment; c – transformed section; d – initial strain distribution

The section contains one or more levels of prestressed reinforcement and is subjected to a bending moment with the top fibre taken as the reference level (Fig 2, a, b). The cross section analysis is performed on the so-called transformed section. The area of the prestressed reinforcement (if it is bonded) is transformed into an equivalent area of concrete (Fig 2, c).

The top fibre strain and curvature are obtained from the following expressions:

$$\varepsilon_{ct} = \frac{-B_e(M - Pe) - I_e P}{\left(A_e I_e - B^2\right)E_c}, \quad (11)$$

$$\kappa = \frac{-A_e(M - Pe) + B_e P}{\left(A_e I_e - B^2\right)E_c}. \quad (12)$$

Here A_e is the area of the transformed section; B_e and I_e are the first and second moments of the area about the top surface; P is the initial prestressing force; M is the applied bending moment; e is the distance from the initial prestressing force to the top surface.

The change of the stress in the prestressed reinforcement after transfer is obtained from the equation:

$$\Delta\sigma_{spi} = E_{spi}\left(\varepsilon_{ct} + y_i\kappa\right). \quad (13)$$

Here y_i is the distance of the i-th layer from the top edge.

A computer program has been developed for assigning curvatures and deflections and the average stress and strain state at any point of the member. The procedure for both short- and long-term deformational analysis of flexural reinforced concrete members has been described [8–11].

3. Comparison of deflections assessed by different methods with test results

This section compares mid-span deflections assessed by the four methods described in the previous section with test data of 26 prestressed concrete beams reported by Warvaruk and Sozen [12]. The range of main characteristics of the experimental beams such as cross-section parameters, effective prestress, reinforcement ratio and concrete strength are presented in Table 1. All the beams had a rectangular section and were subjected to a short-term loading of two concentrated forces which divided the beam into three equal pieces. Beam deflections were calculated at five load levels, ie 40, 55, 60, 70 and 80 % of the experimental ultimate moment, M_u.

Accuracy of the predictions made by each method was assessed by statistical parameters such as the mean value and the standard deviation calculated for relative deflections $f_{th}/f_{\exp}$ were f_{th} and $f_{\exp}$ are the calculated and the experimental deflections respectively. These statistical parameters assessed for the predictions by the methods of the present analysis, PCI [1], Eurocode [2], and the Russian Code [3] are presented in Table 2. It

Table 1. Main characteristics of beams

No	Characteristics of data	Total number of beams	Span, [m]	Height, [m]	Width, [m]	Effective pre-stress, [MPa]	Reinforcement ratio, [%]	100 mm cube strength, [MPa]
1.	Prestressed beams	26	3.05	0.305-0.307	0.153-0.158	132-1044	0.109-0.943	18.00-45.20

Table 2. Statistical parameters for relative deflections, $f_{th} / f_{\exp}$, estimated by different methods

No	Characteristics of data	PCI		EC2		SNiP		Present analysis	
		Mean	Stand.	Mean	Stand.	Mean	Stand.	Mean	Stand.
1.	0,4 M_u (26 deflection points)	0,939	0,141	0,870	0,132	0,989	0,104	0,952	0,081
2.	0,55 M_u (26 deflection points)	0,960	0,191	0,994	0,236	1,007	0,156	0,945	0,134
3.	0,6 M_u (26 deflection points)	0,962	0,155	0,963	0,227	1,004	0,135	0,940	0,126
4.	0,7 M_u (26 deflection points)	0,945	0,154	0,948	0,212	1,008	0,134	0,958	0,117
5.	0,8 M_u (26 deflection points)	0,874	0,144	0,871	0,198	0,951	0,161	0,951	0,147
6.	Total (130 deflection points)	0,935	0,161	0,920	0,207	1,040	0,175	0,950	0,124

contains results for each relative moment level corresponding to 40, 55, 60, 70 and 80 % of M_u and the results for the total data. The most accurate predictions were obtained for the methods of the present analysis, PCI and the Russian Code giving the standard deviation 12,4, 16,.1 and 17,5 %, respectively (Table 2). The EC2 method gave an error of 20,7 %.

4. Concluding remarks

In this paper, an attempt has been made to extend the application of the Flexural constitutive model to a short-term deformational analysis of flexural prestressed concrete members.

Accuracy of the earlier proposed constitutive relationship for the tensile concrete in flexure has been investigated by means of deflection estimation of 26 experimental prestressed concrete beams.

Comparison with the experimental deflections at five load levels and with the estimates of three other methods has been performed. Accuracy of the predictions has been assessed by statistical parameters such as the mean value and the standard deviation calculated for relative deflections.

The best agreement in terms of standard deviation assessed for the total data has been achieved for the present analysis, the PCI and SNiP methods (12,4, 16,1 and 17,5 %, respectively). The EC2 method gave an error of 20,7 %. The results have shown that the Flexural constitutive model is applicable for the deformational analysis of prestressed beams.

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