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# HIGH-STRENGTH CONCRETE HOLLOW BEAMS STRENGTHENED WITH EXTERNAL TRANSVERSAL STEEL REINFORCEMENT UNDER TORSION

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**Abstract.** Some bridges have to withstand high levels of torsion forces. As a consequence, box type beams are often the obvious solution. It could be possible that the balance of transversal to longitudinal torsion reinforcement is not fully reached. If the transversal reinforcement is somehow underestimated, the box beam needs to be transversally strengthened. From the various solutions, external transversal reinforcement is certainly one possibility. The investigation presented here aimed to study such solution. The authors tested four hollow beams under pure torsion. The level of the non balanced ratio between internal longitudinal and transversal torsion reinforcement was one of the parameters that were considered in this investigation. Other parameter was the existence or the no existence of external transversal strengthening reinforcement. The experimental results of the tests have shown the effectiveness of the use of the external transversal strengthening steel reinforcement to compensate the lack of balance of internal transversal to longitudinal torsion reinforcement with respect to various behaviour aspects, such as: increasing of torque strength, increasing of ductility, increasing of cracking torsion moment, and better distribution of cracking.

**Keywords:** RC beams, torsion design, bridges, hollow beams, strengthening of structures, high strength concrete.

## 1. Introduction

In the last century a great number of infrastructures were constructed, including many bridges. The existence of important torsion forces normally conducted to hollow beams. A correct balance between longitudinal and transversal torsion reinforcement is important for the structural performance with respect to torsion actions. Such balance is not always achieved in design. Beyond this, the aging of the structure might introduce some further defects to the structure. To correct the defects, repair works are needed. This might include the strengthening of the structures. In fact, the number of repair works in bridges is increasing in the last years. Some unexpected failures of bridges pressed the public opinion and the leading politicians to pay attention to the problem of degradation of such structures. The increment of the service actions and traffic level, the reduction of the strength capability to environmental actions or accidents and the new design rules related with earthquake actions are other aspects which might be associated with the degradation of the structures and need to be considered when evaluating the need of repair of a structure.

The advantages of using high-strength concretes (HSC) in a great number of special constructions (bridges, for example), where the demands for durability and serviceability performance are high, have been persuaded some design engineers and construction companies. The use of HSC can lead to cheaper structures due to the possibility of using smaller sections and thicknesses in reinforcement elements. The overall weight of the structures is reduced, having the inherent advantages from the earthquake behaviour point of view. This way, the experience has shown that the advantages of the HSC, comparatively to normal strength concretes (NSC) compensate many times the higher unity cost of such material, taking into account that a careful selection of the aggregates, a good mix design, curing and quality control are essential for not spoiling such advantages.

Even in the case of bridges (straight or curved), pure torsion hardly occur in actual structures. Torsion is normally combined with other forces. However, the general design by considering the interaction of forces includes a first computation of forces in pure torsion. Therefore, the study of the elements under pure torsion is an important issue.

No matter which strengthening technique is used, the knowledge about the strengthened beams under torsion forces is very small when compared with beams under bending and shear. Because of this, due to the lack of available technical documentation, it is common nowadays to use existing information specific to the transversal strengthening to solve problems related to torsion. Many bridges are subjected to significant torsion strength that conditions the sizing and may require reinforcement. That justifies the importance of the theme analyzed in this paper.

Since the construction of bridges is a typical example of the use of HSC and hollow beams correspond to a normal solution for bridges decks, research works on the behaviour of high-strength concrete hollow beams under torsion are of great interest.

This paper aims at studying the global behaviour of HSC hollow beams under pure torsion and the possibility of transversal reinforced made with external transversal steel bars. A set of four experimental tests have been carried out in laboratory.

## 2. Overview and significance of research

There are little experimental studies of HSC beams under pure torsion. Nevertheless, some studies deserve to be mentioned. For instance, Rasmussen and Baker (1995a, b) and Wafa *et al.* (1995) published studies on torsion and the general analyses of the results of those studies revealed some of the advantages of the use of HSC, mainly on the structural behaviour after cracking and on the maximum torque. Bernardo and Lopes (2008, 2009b) and Lopes and Bernardo (2009) published some torsion studies on NSC and HSC hollow beams.

Beyond the positive aspects that have already been cited, these authors have alerted for some disadvantages of the use of HSC from the point of view of the torsion ductility. Normally, ductility is not related with torsion forces, but the authors have shown that NSC hollow beams can show some ductility.

Obviously, ductility is more easily found in beams under flexure. There are many works on the ductility and plastic rotation capacity of reinforced concrete beams under flexure. Examples of this are the works by Bernardo and Lopes (2004, 2009a), Bręstrup (2000), Carmo and Lopes (2005, 2008), Lopes *et al.* (1997), Lopes and Bernardo (2009) and Lopes and Carmo (2006). In general, HSC members are less ductile than NSC members.

The subject of shear forces associated with ductility was studied by Jensen and Lapko (2009). Kliukas *et al.* (2010) reported a study on ring beam-columns under bending and compressive forces. They found that high strength reinforcement steel led to higher ductility when compared with normal strength reinforcement steel.

Despite the studies on torsion behaviour of hollow beams that were already published, the number of results of tests is still very limited and more studies are needed to support a firm theory on the behaviour of such structures.

The authors did not find any scientific article dealing with strengthening of HSC hollow beams under torsion. There are some studies on NSC beams and most of them are on plain beams. With this respect, some articles deserve to be mentioned. This is the case of the works by Ameli *et al.* (2004), Ghobarah *et al.* (2002), Hii and Al-Mahaidi (2004, 2005), Panchacharam and Belarbi (2002), Salom *et al.* (2004) and Zhang *et al.* (2001). In such studies there is only one strengthening technique covered: carbon fibres sheets laminates.

For HSC the type of the $\sigma$–$\varepsilon$ relationship is different when compared with that of NSC. This is clearly explained by Dahl (1992). The HSC is more fragile than NSC and this has consequences when the concrete is used in structures. The structural behaviour of HSC members is also more fragile than that of NSC. The reduction of ductility in members under flexure is not problematic in many cases. However, for beams under torsion, the reduction of ductility when passing from NSC to HSC is very sharp. These aspects are explained in previous articles by the authors, namely, Bernardo and Lopes (2008, 2009b) and Lopes and Bernardo (2009). As a consequence, it seems understandable that, if a HSC member under torsion needs to be re-strengthened, the choice of the strengthening material might have in mind the ductility characteristic of such material.

From the explained before, carbon fibres sheets laminates might not be adequate for strengthening HSC beams under torsion, due to the fragile behaviour of such materials. Furthermore, this is an expensive technique. These reasons justify why such strengthening technique has not been used in this study. A more traditional strengthening technique has been chosen, external steel bars braced around the section to form a kind of external stirrups.

In the case of big hollow sections, like those that are used in bridges, the reinforcement can be used around the cross section or put inside the void of section. This solution could be very interesting if aesthetical considerations are important.

This study aims at evaluating experimentally the effectiveness of a transversal strengthening in hollow HSC beams under torsion, when a strong deficit (in volumetric terms) of the transversal reinforcement compared to the longitudinal reinforcement needs to be compensate. In fact, in beams under high levels of torsion forces, such deficit would easily lead to a fragile failure due to insufficient transversal reinforcement.

## 3. Test Program

### 3.1. Test specimens

Four HSC hollow beams were tested for this investigation. The beams, 5.90 m long, had a square section and were tested up to failure. The beams were put in a test apparatus which restricted the torsion in one of the ends and applied a torsion moment on the opposite end. Fig. 1 illustrates the general geometry of the beams (constant for all the specimens) as well as the arrangement of the internal and external reinforcement. The beams had a square void. The exterior dimension of the sides were 0.60 m and  the wall was 0.10 m thick, being close to the shape of the section that other researchers, namely Lampert and Thurlimann had adopted for their experiments, as reported in CEB (1969).

The dimensions of the section were also defined taking into account an adequate scaling, the transportation restrictions (the beams were constructed in a precast plant and then transported to the lab) and the specifications of the actuator. Due to the global dimensions of the models, the transversal reinforcement bars were assembled externally. The ends of the beams had a geometry that fitted in the steel parts of the testing apparatus. The concentration of stresses at the ends of beams was taking into account and the beams were designed to fail far from these zones.

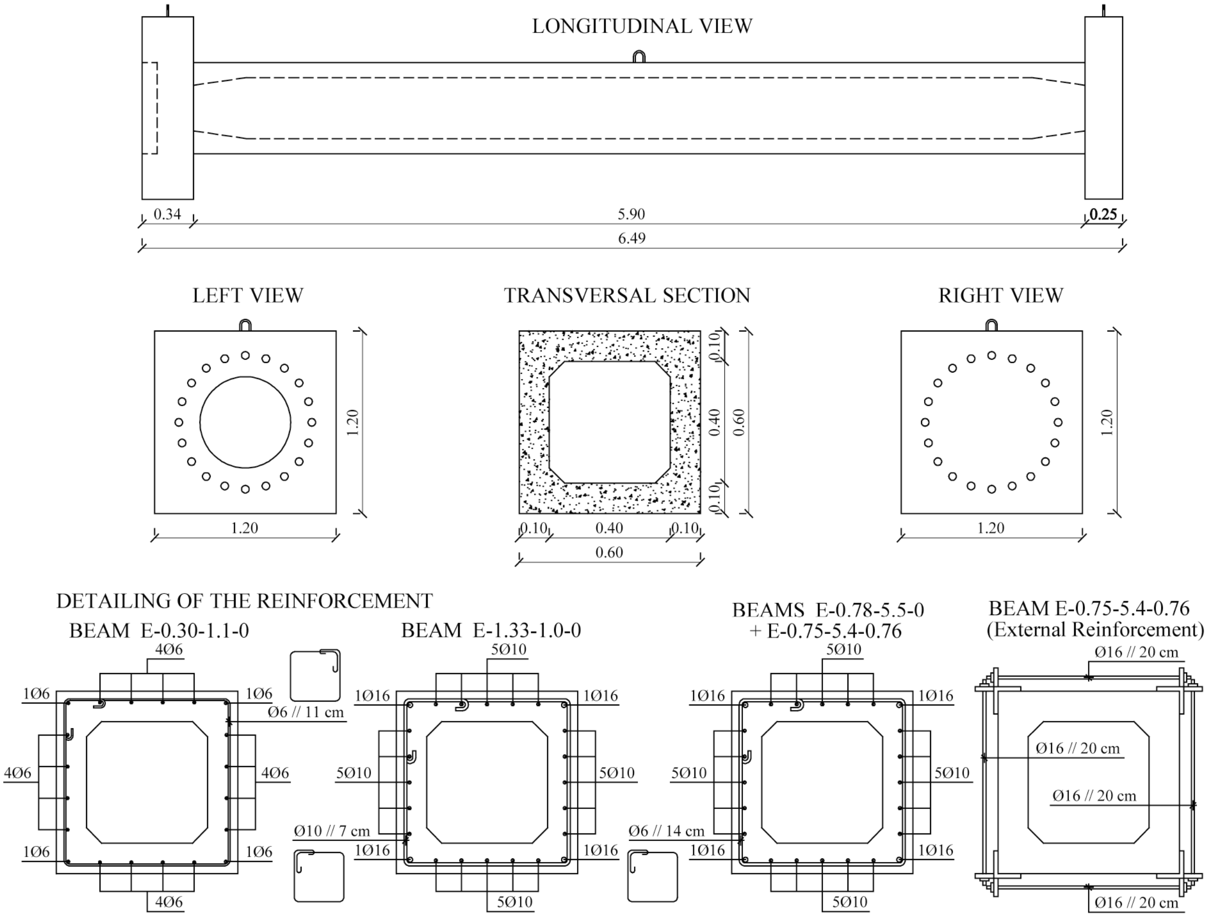


**Fig. 1.** Geometry and detailing of tested beams

**Table 1.** Properties of the tested beams

| Beam | $t$ cm | Long. reinf. | Transv. reinf. / ext. transv. reinf. ($\phi@s$) | $x_1$ / $y_1$ cm | $A_{sl}$ / $A_{st}$ cm$^2$ | $\rho_l$ / $\rho_t$ / $\rho_{tot}$ / $\rho_{t,ext}$ % | $m_b$ | $f_c$ / $f_{ctm}$ MPa | $E_c$ GPa |
|---|---|---|---|---|---|---|---|---|---|
| E-0.30-1.1-0 | 10.1 | 20$\phi$6 | $\phi$6@11cm / – | 53.9 / 54.4 | 5.65 / 0.28 | 0.16 / 0.14 / 0.30 /– | 1.11 | 75.6 / 4.26 | 40.4 |
| E-1.33-1.0-0 | 10.9 | 4$\phi$16+ 20$\phi$10 | $\phi$10@7cm / – | 53.5 / 53.7 | 23.75 / 0.79 | 0.66 / 0.67 / 1.33 /– | 0.99 | 77.8 / 4.33 | 40.7 |
| E-0.78-5.5-0 | 10.7 | 4$\phi$16+ 20$\phi$10 | $\phi$6@14cm / – | 54.0 / 53.6 | 23.75 / 0.28 | 0.66 / 0.12 / 0.78 /– | 5.52 | 78.0 / 4.34 | 40.8 |
| E-0.75-5.4-0.76 | 10.9 | 4$\phi$16+ 20$\phi$10 | $\phi$6@/14cm/ $\phi$16@20cm | 54.0 / 55.3 | 23.75 / 0.28 | 0.66 / 0.12 / 0.75 / 0.76 | 5.44 | 80.5 / 4.42 | 41.1 |

The test variables that were considered were the ratio between transversal and longitudinal reinforcements of torsion (internal reinforcement), as well as the presence or non existence of external transversal strengthening. The concrete strength, varied between 75.6 and 80.5 MPa. The beams without transversal reinforcement deficit were designed respecting the principle of balanced transversal and longitudinal reinforcement. In general, the amount of reinforcement and the detailing follow the rules presented in European and American standards or codes of practice. The beams with an imposed transversal reinforcement deficit had an amount of transversal reinforcement close to the limitation presented by the American ACI code. The authors read various codes, and ACI code is the only one that has a specific equation for the computation of minimum amount of torsion reinforcement.

Table 1 presents the characteristics of each test beam, namely: real measured value of the thickness of the section wall ($t$), the adopted solutions for the torsion reinforcements (including the external transversal reinforcement), distance between centres of branches of stirrup ($x_1$ and $y_1$), area of longitudinal internal reinforcement ($A_{sl}$) and of a branch of the transversal internal stirrup ($A_{st}$), compressive strength of concrete ($f_c$), longitudinal and transversal reinforcement rates ($\rho_l = A_{sl} / A_c$ with

$A_c = x.y$ and $x = y = 60$ cm) ($\rho_t = A_{sl}.u \ / \ A_c.s$ with $u = 2.(x_1 + y_1)$ ), the total reinforcement rate ($\rho_{tot}$) and the ratio between the "volume" of the internal longitudinal to transversal reinforcement ($m_b = A_{sl}.s \ / \ (A_{st}.u)$).

The beams are named accordingly to the group which they belong to (Series E), the total rate of torsion reinforcement (first number), the parameter that measures the volumetric ratio of internal reinforcement (second number), and the external reinforcement transversal reinforcement rate (third number), which correspond to (E – $\rho_{tot}$ – $m_b$ – $\rho_{t,ext}$).

The Beam E-0.78-5.5-0 has a great deficit of transversal internal reinforcement. The Beam E-0.75-5.4-0.76 is very similar to the first one, except for having a transversal reinforcement constituted by external stirrups placed 20 cm apart. Such stirrups were computed to compensate the initial deficit of transversal torsion reinforcement.

### 3.2. Material properties

The compressive strength of concrete was obtained by testing 15 cm cubes under compression. A set of five cubes were tested at the same day of each test of the beams. The final value of the compressive strength of concrete was referred to cylindrical specimens, $f_{ctm}$, by following the guidelines for HSC given by Taerwe (1996). These guidelines have also been used to compute the tensile strength and the modulus of elasticity of concrete correlating them with the compressive strength of concrete. These values can also be found in Table 1. The concrete mix is presented in Table 2.

**Table 2.** Mix design of concretes

| Components | Mix design (/m$^3$) |
|---|---|
| Thin sand (kg) | 164 |
| Thick sand (kg) | 908 |
| Crushed Granit 5/11 (kg) | 734 |
| Normal Portland Cement Type I/42.5R (kg) | 375 |
| Admixture – Rheobuild 1000 (l) | 4.8 |
| Silica Fume (Sikacrete HD) (kg) | 41 |
| Water (l) | 145 |

S500 Class of commercial ribbed rods with diameters that vary between 6 and 16 mm were used as reinforcement. Six specimens of each diameter were tested and the yield strength and the correspondent strain were respectively, $f_y$ = 686 MPa and $\varepsilon_y$ = 3430 μ. S400 Class, 16 mm wide bars were used for the external stirrups. Six specimens of each diameter were tested and the yield strength and the correspondent strain were respectively, $f_y$ = 575 MPa and $\varepsilon_y$ = 2876 μ.

### 3.3. Testing procedure

The test apparatus is formed by a frame test to which an actuator is fixed. A torsion device receives the force from the actuator and applies a torsion moment on the beam (Fig. 2).

Fig. 2 shows a plan and an elevation of the global test apparatus with a beam in its position. The whole structure is fixed to the strong slab of the lab. The coupling of the beams to the apparatus was conceived to not restrict some bend of the cross section at the end of beams neither the beams' axial deformations.

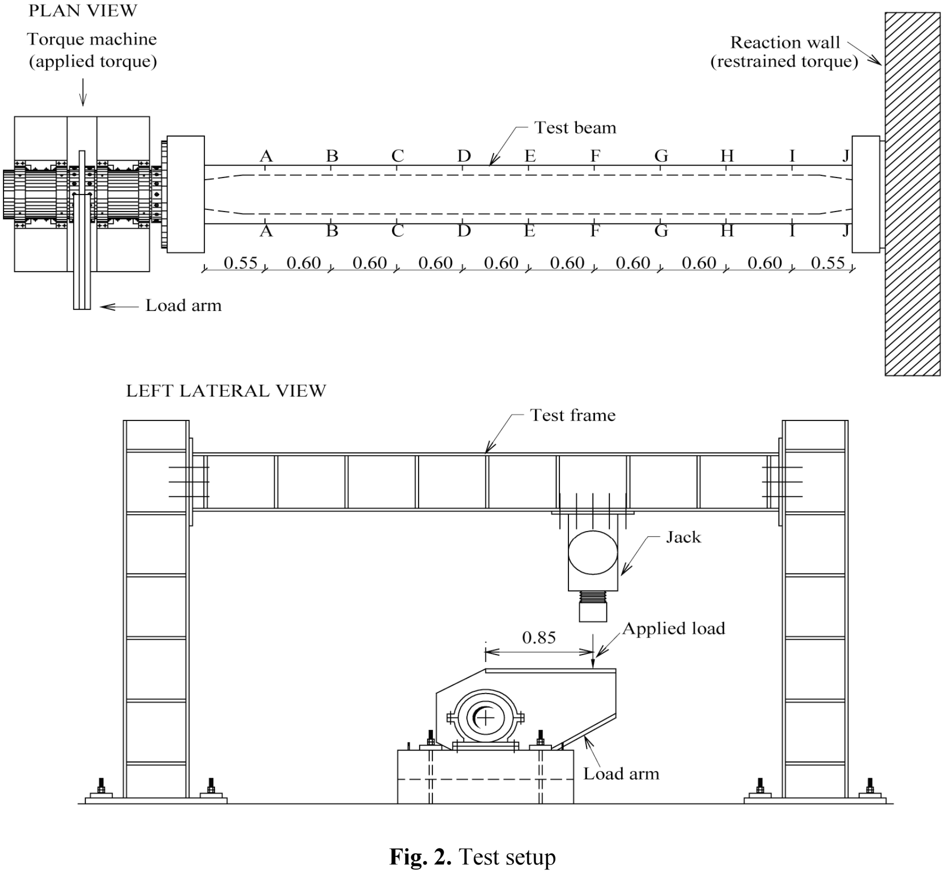


**Fig. 2.** Test setup

The anchorage of the external stirrups to Beam E-0.75-5.4-0.76 is done in the corners of the concrete section, by means of a cross-shaped steel part, which is adjustable to the geometry of the corner and has a pair of holes for the bars to pass through (Fig. 3a). The final work of the assembly is shown in Fig. 3b.

a)

b)

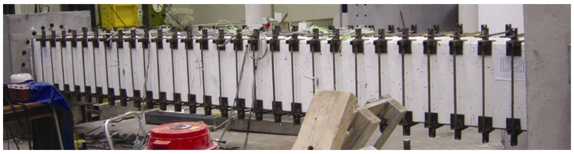

**Fig. 3.** Transversal Reinforcement of Beam E-0.75-5.4-0.76: a) assembly; b) final work

Testing of the beams was carried out by control of deformations and the post peak behaviour could be also monitored.

To control the loading state at any time, several load cells were placed in key locations of the global test apparatus.

The rotations have been read in 10 cross sections of the beams covering the whole length (at Sections A–A to J–J), as it is shown in Fig. 2. For that, 10 pairs of displacement transducers were put on the top side of the beam, and were fixed to external points. This procedure permitted to compute the rotation angles in each section from the readings of the transducers.

The torsion reinforcement bars inside the concrete beam were instrumented in three sections placed at quarters of the total length of the beams. In such sections, strength gauges were pasted to 4 longitudinal bars at corners and to 4 branches of the transversal reinforcement. In the reinforcement external bars no strength gauges were used.

Data gathered by the measuring devices put on the beams and on the global test apparatus were recorded by data logger and transferred afterwards to normal PCs. Fig. 4 shows an overview of a beam (without transversal reinforcement) in its test position with the measuring devices ready for data recording.

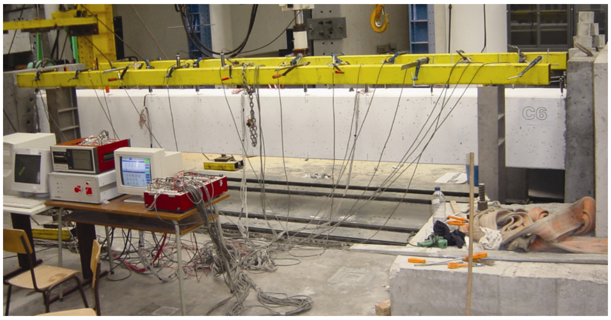

**Fig. 4.** Beam specimen in test position

## 4. Global analysis of the experimental results

### 4.1. Evolution of torque with angular deformation

Fig. 5 presents the torque (T) versus angular deformation ($\theta_m$) graphs of the tested beams. The torque, T, results from the load applied to an arm of the torsion device with an eccentricity of 0.85 m with relation to the centre of rotation of this torsion device (Fig. 2). The average angular deformations of the beams $\theta_m$, (average twist) is obtained by dividing the differences of the angles measured in Sections A–A and J–J by the distance between them, 5.35 m (Fig. 2). In each $T$–$\theta_m$ curve, the cracking points (O) and the points corresponding yielding of transversal ("□") and longitudinal ("Δ") internal reinforcement are identified. The identification of the yielding points of the reinforcement bars was possible through the strain gauges suck to those bars.

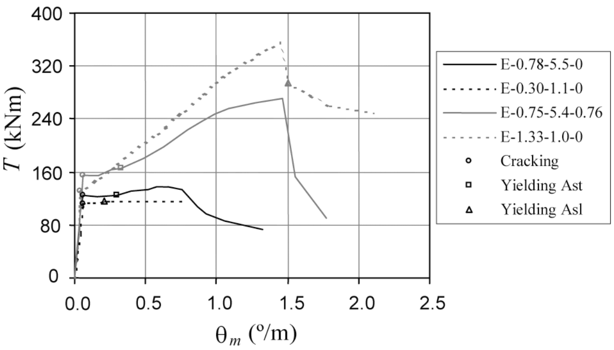


**Fig. 5.** $T$–$\theta_m$ curves

The first conclusion that could be drawn from Fig. 5 is related with the influence of the deficit of transversal reinforcement on the behaviour of the beam. Beams E-0.30-1.1-0 and E-0.78-5.5-0 are very similar, except for the longitudinal reinforcement rate. While the first beam has a reinforcement which is almost balanced with $m_b = 1.11$ (Table 1), the second beam has a strongly unbalanced longitudinal to transversal reinforcement with $m_b = 5.52$. Consequently, in volumetric terms, this second beam presents an exaggerated amount of longitudinal reinforcement compared to the amount of transversal reinforcement (as said before, around the minimum value, as indicated by ACI). Fig. 5 clearly shows that the curves of both beams are somehow similar. Since the transversal reinforcement rates of both beams are similar, the extra amount of longitudinal reinforcement of Beam E-0.78-5.5-0 when compared to that of Beam E-0.30-1.1-0 does not produce a significant impact on the torsion behaviour of the beam. The short increasing on the strength observed may be justified by the fact that, in this beam, cracking occurs slightly later due to the excess of longitudinal reinforcement. Indeed, it is known that the quantity of reinforcement influences the cracking torque, as explained by Hsu (1984). Both beams have the same behaviour after cracking, with the quick yielding of the longitudinal and transversal reinforcement (with the exception of the longitudinal reinforcement of Beam E-0.78-5.5-0 because of the "excess" of such reinforcement). Both beams had a fragile failure due to insufficient reinforcement because the torsion strength does not increase significantly after the cracking torsion moment. This type of behaviour confirms other studies that indicate the non-effectiveness of the longitudinal reinforcement to torque when performing alone (without transversal reinforcement). Hsu (1984) defend that for effective torsion strength the beam must have both types of reinforcement in a balanced "volume". As shown in this investigation this aspect is also valid for HSC beams (and not only for NSC beams).

Fig. 5 also shows that adding external transversal reinforcement to a beam with a deficit of internal transversal reinforcement, significantly improves the behaviour under torsion of such beam. Beam E-0.75-5.4-0.76 is similar to Beam E-0.78-5.5-0 except for the fact that it had been transversally reinforced by using external stirrups. The amount of such stirrups, as mentioned before, was computed by trying to compensate the deficit of the transversal reinforcement. The authors have considered the hypothesis that external transversal reinforcement would produce the same structural consequences as the same amount of internal transversal reinforcement.

Fig. 5 shows that the strengthened beam, Beam E-0.75-5.4-0.76, has a higher cracking moment, when compared to Beam E-0.78-5.5-0. The external stirrups, due to the fact that they were not bonded to concrete, do not increase by themselves the cracking moment. The substantial delay of cracking may be justified by the confinement effect caused by the procedure that was used to assembly the stirrups. A certain level of prestress was used and this would have some influence on the cracking moment of the beam. The torsion stiffness of Beam E-0.75-5.4-0.76 in State I (non cracked phase) seems not to be affected by the exterior stirrups.

When comparing with the non-strengthened beam (Beam E-0.78-5.5-0), the external stirrups of Beam E-0.75-5.4-0.76 is visibly effective after cracking (in State II), since the beam shows a substantial torque increase in this phase. Such increase is noticed even after the yield of the internal transversal reinforcement that occurs immediately after the cracking of the beam. The participation of the external transversal reinforcements at the post-cracking moment may be considered in two ways. First, after beam cracking, the external transversal reinforcements are effectively mobilised for the resistance to torsion. Second, if the external stirrups become participative, they will be equivalent to common transversal reinforcement, and, therefore, compensate the deficit of internal transversal reinforcement. As a consequence, the initial excess of longitudinal reinforcement also becomes effective, and will work together with the total transversal reinforcement (including the external percentage). This behaviour is confirmed by the records of strength gauges glued to the longitudinal reinforcement bars. The measured strains showed that, contrarily to what happened during the test of the beam without external stirrups (Beam E-0.78-5.5-0), during the test of the beam with external stirrups the longitudinal bars start working actively at post-cracking phase (see next section). Fig. 5 shows that the use of the external reinforcement in Beam E-0.75-5.4-0.76 transform a beam from being an example of non recommended solution (fragile failure due insufficient reinforcement) to an example of a suitable solution (the type of failure shows some ductility). This aspect shows that a good potential of the technique of transversal strengthening of a beam with great deficit of transversal reinforcement.

It is also important to do a comparative analysis between Beam E-0.75-5.4-0.76 and Beam E-1.33-1.0-0. The last one is very similar to Beam E-0.75-5.4-0.76, except for the fact that the internal reinforcements are balanced. Beam E-1.33-1.0-0 respects the principle of balanced volume of transversal to longitudinal reinforcement. Fig. 5 clearly shows that, although the external stirrups used in Beam E-0.75-5.4-0.76 are very effective, they do not quite replace the same amount of internal transversal reinforcement. This conclusion comes from the fact that Beam E-1.33-1.0-0 (with balanced internal longitudinal to transversal reinforcement) shows a better behaviour in State II comparatively to Beam E-0.75-5.4-0.76. This difference may be justified by the lack of bonding between the external reinforcement and concrete. After cracking, Beam E-0.75-5.4-0.76 shows a torsion stiffness value lower than that of Beam E-1.33-1.0-0. As far as the maximum torque is concerned, an important difference can be found when compared both beams. Beam E-1.33-1.0-0 achieves a much higher value of the strength peak. Therefore, the strengthening technique used in this work is not entirely effective after cracking, when compared to an equivalent and normal beam, as, for example, Beam E-1.33-1.0-0. The design of this kind of reinforcement must be carried out with care and designing rules need to be investigated. Nevertheless, the strengthening technique introduces remarkable improvements on the structural behaviour of the beams and deserves to be further studied.

Table 3 presents the summary of the characteristics of $T$–$\theta_m$ curves shown in Fig. 5, namely; cracking torsion moment and its correspondent rotation ($T_{cr}$ e $\theta_{cr}$), torsion stiffness in State I ($(GC)^{\mathrm{I}}$), torsion stiffness in State II ($(GC)^{\mathrm{II}}$), torsion moment correspondent to the beginning of the yield of the internal transversal reinforcement and respective rotation ($T_{ty}$ and $\theta_{ty}$), torsion moment correspondent to the beginning of the yield of the longitudinal reinforcement and respective rotation ($T_{ly}$ and $\theta_{ly}$), torque strength (maximum) and respective rotation ($T_r$ and $\theta_{Tr}$).

**Table 3.** Characterization of $T$–$\theta_m$ Curves

| Beam | $T_{cr}$ (kNm) | $\theta_{cr}$ (°/m) | $(GC)^{I}$ (kNm²) | $T^{II} = a\theta^{II} + b$ | $(GC)^{II}$ (kNm²) | $T_{ty}$ (kNm) | $\theta_{ty}$ (°/m) | $T_{ly}$ (kNm) | $\theta_{ly}$ (°/m) | $T_r$ (kNm) | $\theta_{Tr}$ (°/m) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| E-0.30-1.1-0 | 111.5 | 0.06 | 107198 | a = 10.69<br>b = 113.05 | 612 | – | – | 115.4 | 0.21 | 116.0 | 0.23 |
| E-1.33-1.0-0 | 130.5 | 0.04 | 172940 | a = 170.87<br>b = 119.43 | 9790 | – | – | – | – | 355.9 | 1.45 |
| E-0.78-5.5-0 | 124.54 | 0.056 | 126592 | a = 43.17<br>b = 112.10 | 2474 | 124.2 | 0.30 | – | – | 136.9 | 0.65 |
| E-0.75-5.4-0.76 | 155.56 | 0.064 | 139063 | a = 68.07<br>b = 144.04 | 3900 | 165.3 | 0.32 | – | – | 271.3 | 1.46 |

The torsion stiffness in State I is calculated by dividing $T_{cr}$ by $\theta_{cr}$. To compute the torsion stiffness in State II, the equation of the straight which expresses the behaviour of $T$–$\theta_m$ curves in the linear elastic phase in State II must be computed first by linear interpolation. To do this, the points of the graphs of this phase are selected (only in the zone where the line can be approximate to a straight). After the calculation of the equation, $T = a\theta + b$ the stiffness $(GC)^{II}$ comes equal to slope $a$ of the equation.

The analysis of the values on Table 3 confirms the tendencies observed in Fig. 5 and explained before.

### 4.2. Evolution of strains in reinforcement with twist

As an example, Fig. 6 presents some graphs which refer to the evolution of the extension in longitudinal and transversal internal reinforcement bars ($\varepsilon_{st}$ and $\varepsilon_{sl}$) "versus" angular deformation (twist) ($\theta_m$).

The horizontal zones at the beginning of the $\varepsilon_s$–$\theta_m$ curves are explained by the fact that the bars are not stressed before the cracking of the beams.

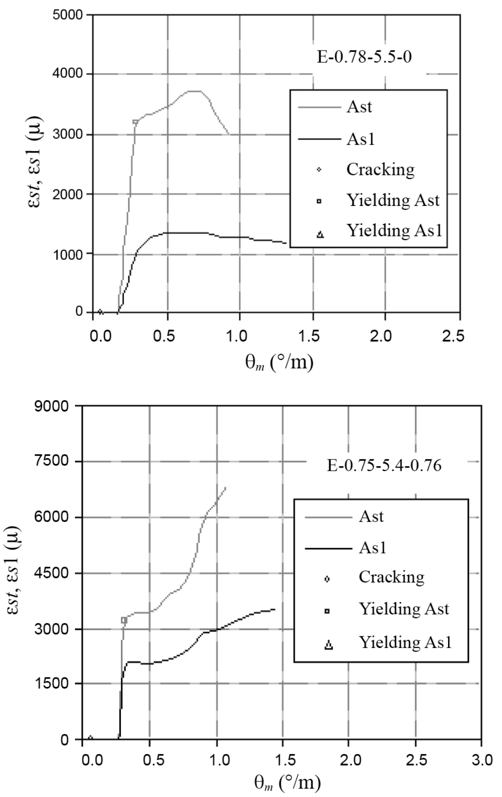


**Fig. 6.** $\varepsilon_{st}$, $\varepsilon_{sl}$ –$\theta_m$ curves

The $\varepsilon_s$–$\theta_m$ curves of Beams E-0.78-5.5-0 and E-0.75-5.4-0.76 clearly show a sudden increase of deformation in transversal reinforcements after cracking, immediately followed by yield. That sudden increase is compatible with the fact that the transversal reinforcement is much lower, in volumetric terms, to the longitudinal reinforcement. For Beam E-0.78-5.5-0, such behaviour is also compatible with the fact that the failure was very similar to the typical failure by insufficient reinforcement, as it has been shown in Fig. 5.

Fig. 6 shows great differences of the level of stresses of the transversal reinforcement when compared to those of the longitudinal reinforcement (being the stresses much higher in the transversal reinforcement). In the case of Beam E-0.75-5.4-0.76, such differences were not as high as in the Beam E-0.78-5.5-0. This might be due to the existence of external stirrups, which provoked a more balanced situation between transversal and longitudinal reinforcement.

### 5. Analysis of the failure mode

Figs 7–10 show some photos with the failures of the tested beams. The different types of failure described before are visible in these figures.

Fig. 7 illustrates the fragile failure due to insufficient reinforcement of Beams E-0.78-5.5-0 and E-0.30-1.1-0. The failure is characterized by a large helicoidal shaped single crack.

a)

b)

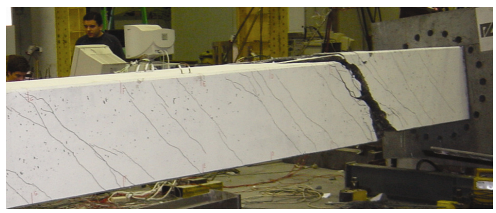

**Fig. 7.** Fragile failure due to insufficient reinforcement:
a) Beam E-0.30-1.1-0; b) Beam E-0.78-5.5-0

Fig. 8 shows a failure by corner break off. This kind of failure, which has a fragile and premature nature, typical of beams with hollow sections, has already been observed and analyzed by Bernardo and Lopes (2008, 2009), and Lopes and Bernardo (2009).

Beam E-0.75-5.4-0.76 (Figs 9 and 10) show a type of failure that was somehow fragile as expected by observing the corresponding $T$–$\theta_m$ curve (Fig. 5).

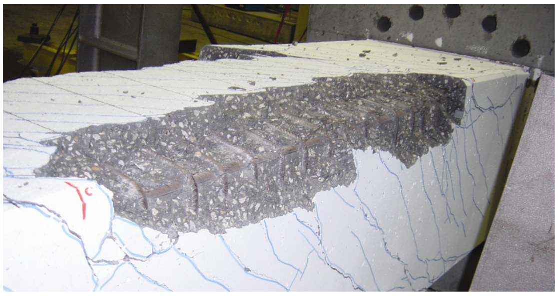

**Fig. 8.** Fragile failure due to corner break off: Beam E-1.33-1.0-0

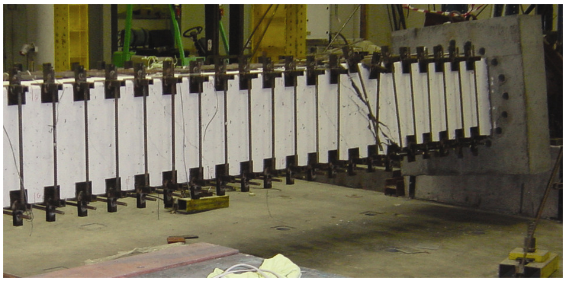

**Fig. 9.** Failure of the strengthened beam: Beam E-0.75-5.4-0.76

**Fig. 10.** Failure of Beam E-0.75-5.4-0.76 cleared from external stirrups

## 6. Cracking pattern analysis

To analyse the cracking pattern of the beams after the tests, the cracking of the beams has been registered on the three visible faces (side and upper faces).

Figs 11–14 illustrate the state of cracking of the tested beams along their length. The black areas represent the cracking and the detachment of some concrete portions from the surface of the beams.

Fig. 13, referring to Beam E-0.78-5.5-0, clearly shows that, although there is a great reinforcement unbalance responsible for a typically fragile failure due to insufficient transversal reinforcement, the longitudinal reinforcement can, even so, have some effectiveness in controlling the cracking. Indeed, the cracking final pattern of Beam E-0.78-5.5-0 reveals a better distribution of the cracks comparatively to Beam E-0.30-1.1-0, which

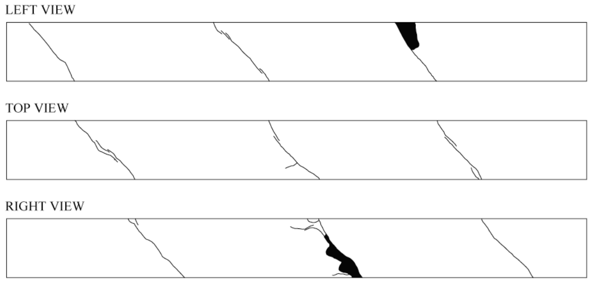


**Fig. 11.** Cracking pattern: Beam E-0.30-1.1-0

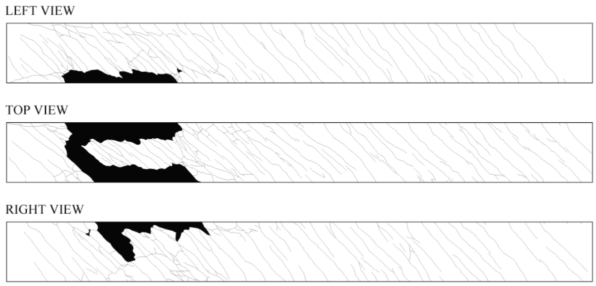


**Fig. 12.** Cracking pattern: Beam E-1.33-1.0-0

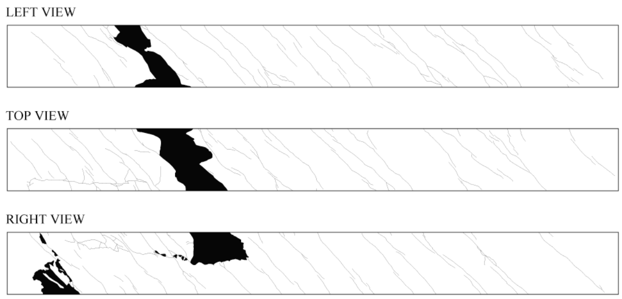


**Fig. 13.** Cracking pattern: Beam E-0.78-5.5-0

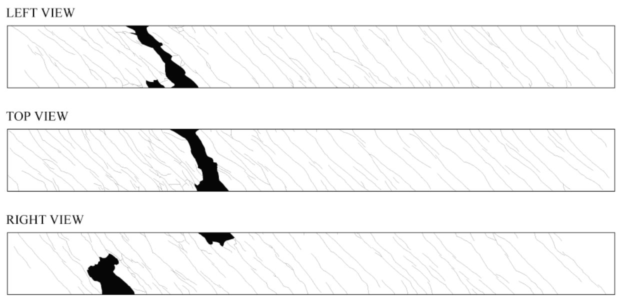


**Fig. 14.** Cracking pattern: Beam E-0.75-5.4-0.76

had a similar transversal reinforcement ratio but with a balanced situation as far as transversal and longitudinal torsion reinforcements are concerned. Therefore, the excess of longitudinal reinforcement could not be of great help in increasing the torsion strength of the beam, but seems to have a positive effect in controlling the cracking.

Fig. 14 refers to Beam E-0.75-5.4-0.76 and shows that, using external transversal reinforcement, cracking can be much better controlled, comparatively to Beam E-0.78-5.5-0 (Fig. 13), although not so well as in Beam

E-1.33-1.0-0 (Fig. 12). Therefore, the external reinforcement technique that has been used for Beam E-0.75-5.4-0.76 is also quite effective in controlling the cracking. The improvement of the distribution of cracking of Beam E-0.75-5.4-0.76 can not be directly associated with the effectiveness of the external stirrups since they are not bonded to concrete. It would be the general confinement effect induced by the external stirrups that improves the cracking control.

Table 4 shows, for each tested beam, the average longitudinal distance between consecutive torsion cracks ($d_m$) and the average angle that the cracks present with relation with the longitudinal axis of the beam ($\alpha_m$).

**Table 4.** Cracking parameters

| Beam | $d_m$ (cm) | $\alpha_m$ (°) |
|---|---|---|
| E-0.30-1.1-0 | 179.3 | 51.2 |
| E-1.33-1.0-0 | 13.0 | 46.3 |
| E-0.78-5.5-0 | 31.2 | 45.1 |
| E-0.75-5.4-0.76 | 16.2 | 45.9 |

The parameter $d_m$ represents the arithmetic average of all the measured distances between the typical torsion cracks, that is, the inclined cracks that cross a face completely. Cracks resulting from the effect of detachment of the corners and secondary or connective cracks between main torsion cracks have been ignored. Therefore, parameter $d_m$ is comparable to the correspondent distance of cracks in beams under bending in Serviceability Limit States analyses.

The angles $\alpha_m$ represent the arithmetic average of all the angles measured between the line of the main cracks at the various faces and the longitudinal axis of the beam at middle-height of the three visible faces.

Table 4 shows that, with exception for Beam E-0.30-1.1-0, the orientation of the cracks remains almost unchanged, being around 45.1° and 46.3°, being very close to the predicted theoretical value of 45° (this would correspond to a beam with balanced reinforcements). Only Beam E-0.30-1.1-0 presents a somehow higher orientation (51.2°). This happens probably because it was the only helicoidal crack of the whole beam. The higher $\alpha_m$ value might correspond to the optimal orientation of the crack to minimise the energy of cracking formation.

The orientation of the cracks seems not to be influenced by the external reinforcement added to the beam.

Table 4 also shows the advantage of using the external transversal reinforcement in Beam E-0.75-5.4-0.76, as reflected on its structural behaviour in State II. By comparing its value of $d_m$ with that of Beam E-0.78-5.5-0, it is clear that the cracking sections are more close to each other, therefore the cracking is more evenly distributed, as mentioned before, when looking at other aspects.

## 7. Conclusions

The experimental results obtained from the tested beams have shown that, for torsion forces, the excess of longitudinal reinforcement in a beam is not effective when the amount of transversal reinforcement is low and in deficit in relation with the longitudinal reinforcement. The behaviour of the beam with unbalanced volume between longitudinal to transversal reinforcement is determined by the amount of transversal reinforcement if this type of reinforcement is in deficit compared with longitudinal reinforcement. The results have also shown that for a beam with deficit of transversal reinforcement, the solution of strengthening with external steel stirrups is very effective and compensates, in great extent, the lack of internal reinforcement. That effectiveness is reflected in the substantial increasing of the cracking torsion moment and of the torsion stiffness in State II. The torque strength of the beam is also significantly increased. However, the effectiveness of the added transversal reinforcement, computed to correct the lack of the internal transversal reinforcement, is not fully satisfactory, since the torsion moments of the strengthened beam did not quite achieve those of a reference beam, which had a balanced situation of the transversal to longitudinal with all reinforcement inside the concrete with no need of external strengthening.

It has also been observed that the excess of longitudinal reinforcement can be somehow beneficial in the control of cracking.

When compared to the beam with a deficit of transversal reinforcement, the use of external stirrups improves the distribution of the cracking, although does not achieve the good behaviour of the reference beam.

Due to the limited number of available experimental results, more experimental studies on the strengthening of HSC hollow beams are needed in order to establish firm design criteria before this technique can be safely used in practice.

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## SUKTINĖS STIPRIOJO BETONO DĖŽINIO SKERSPJŪVIO SIJOS, SUSTIPRINTOS IŠORINE PLIENINE SKERSINE ARMATŪRA

**L. F. A. Bernardo, S. M. R. Lopes**


Santrauka

Kai kurie tiltai turi atlaikyti dideles sukimo jėgas. Tam tikslui dažnai naudojamos dėžinio skerspjūvio sijos. Gali būti, kad tarp sukimui atlaikyti naudojamos skersinės ir išilginės armatūros ne visada pasiekiamas tinkamas balansas. Jei skersinė sija armuota nepakankamai, dėžinio skerspjūvio sijas gali tekti papildomai stiprinti. Vienas iš įvairių galimų stiprinimo variantų – armavimas išorine skersine armatūra. Šiame straipsnyje pateikiama tokio stiprinimo analizė. Autoriai išbandė keturias grynojo sukimo veikiamas dėžinio skerspjūvio sijas. Vienas iš tyrimo parametrų – skersinės ir išilginės sukimo armatūrų santykio nesubalansuotumo lygis. Kitas parametras – išorinės skersinės stiprinimo armatūros buvimas arba nebuvimas. Eksperimentinių tyrimų rezultatai parodė stiprinti naudojamos išorinės plieninės skersinės armatūros veiksmingumą, kompensuojant vidinės skersinės ir išilginės suktinės armatūrų tarpusavio nesubalansuotumą. Efektyvumas buvo įrodytas tokias aspektais: padidėjusi sukamoji galia ir elastingumas, padidėjęs plyšių atsiradimo sukimo momentas bei geresnis plyšių pasiskirstymas.


**Reikšminiai žodžiai:** gelžbetoninės sijos, suktinių sijų projektavimas, kiaurymėtosios sijos, konstrukcijų stiprinimas, stiprusis betonas.


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