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DIAGONAL TENSION FAILURE OF RC BEAMS WITHOUT STIRRUPS

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Abstract. The shear failure of reinforced concrete beams is one of the fundamental problems in civil engineering; however, the diagonal tension strength of reinforced concrete (RC) beams without stirrups is still in question. This paper focuses on the prediction of diagonal cracking strength of RC slender beams without stirrups. In slender beams, flexural cracks develop in the tension zone prior to a diagonal cracking. Using the basic principles of mechanics, but cracking included, and theory of elasticity, a diagonal cracking strength equation is proposed for both normal and high strength concrete beams. The proposed equation, the requirements of six codes of practice and seven equations proposed by different researchers are compared to the experimental results of 282 beams available in the literature. It is found that the predictions from the proposed equation are in good agreement with the experimental results.

Keywords: compressive strength, reinforced concrete, cracking, shear strength, slender beam, diagonal tension.

1. Introduction

The results of experiments show that the shear failure of reinforced concrete (RC) slender beams without stirrups is always governed by diagonal tension failure mode rather than compression failure mode. During the last 40 years, researchers have made several attempts to predict the shear strength of RC beams based on mainly experimental results and statistical studies. With respect to the various empirical formulas, considerable differences exist as a result of the following factors: the uncertainty in assessing the influence of complex parameters in a simple formula; the scatter of the selected test results due to inappropriate tests being considered; the poor representation of some parameters in tests; and finally, the concrete tensile strength often not being evaluated from control specimens. These issues limit the validity of empirical formulas, and increase the necessity for rational models and theoretically justified relationships (ASCE-ACI445 1999). It is believed that an analytical formula is more satisfactory than an empirical formula, as it provides physical insight into the phenomenon (Gastebled, May 2001).

To determine the minimum amount of stirrups and to obtain the shear strength of RC beams with stirrups, it is necessary to know the diagonal cracking shear strength of RC slender beams. In case of a slender beam with shear span-to-depth ratio $a/d > 2.5$, inclined tensile cracking develops in the direction perpendicular to the principal tensile stress axis when the principal tensile stress within the shear span exceeds the tensile strength of concrete. Taylor (1960) has indicated that the diagonal cracking stage is not clearly defined in the experimental beams where the crack formed is close to the applied load because the development of the inclined cracks is gradual. Since the diagonal cracking load is very sensitive to the judgment of the observer and the location of the initiating flexural crack, experimental values scatter significantly (Bazant, Kazemi 1991). Diagonal cracking shear strength is defined in Mphonde and Frantz (1984) tests as the shear load when the critical crack becomes inclined and crosses at mid-depth. The inclined shear strength, therefore, obviously is affected by the observer's judgment and also is sensitive to the actual location of the initiating flexural crack.

Mphonde and Frantz (1984) tests on beams without stirrups have shown that the ratio of the shear causing inclined cracking to the measured shear strength ranges from 0.74 to 0.97 and is very unpredictable. Therefore, it is difficult to determine the value of the diagonal tension stress and the cracking load in a RC beam because the distributions of shear and flexural stresses are not known precisely. Furthermore, the crack initiation load is not proportional to the failure load and it can be much smaller or only slightly smaller depending on the beam size and other factors (Bazant, Kazemi 1991). The contribution of this study is to present an equation for predicting the diagonal cracking strength of RC slender beams without stirrups. The proposed equation, the requirements of six codes of practice and seven equations proposed by different researchers for either cracking or ultimate shear strength are compared to the experimental diagonal cracking shear strengths available in the literature.

To determine the minimum amount of stirrups and to obtain the shear strength of RC beams with stirrups, it is necessary to know the diagonal cracking strength of

RC slender beams. The requirements of several codes and methods of prediction of the shear strength are based on the experimental results of normal strength concrete. In design, using these equations may not be appropriate, and verifications and modifications may be required for the evaluation of shear strength of high strength concrete beams. In this study, using the basic principle of mechanics and calibrating against the factors of the effective depth and slenderness ratio, an equation for predicting diagonal cracking strength is proposed for both normal and high strength concrete beams.

2. Existing shear strength models

A number of equations proposed by various codes and researchers are considered. These are ACI318 Building Code (2008), based on experimental results of numerous beams; Turkish Building Code (TS500 2000), based on the adaptation of ACI Code simplified equation; CSA Code (1994), based on the modified compression field theory; NZS (1995); EN 1992-1-1:2004 (2004); CEB-FIP90 (1993) model code equation, introduced empirically; Zsutty's equation (Zsutty 1971), deduced by multiple regression analysis; Okamura's equation (Okamura, Higai 1980), developed empirical equation from experimental data; Bazant's equation (Bazant, Kim 1984), based on non-linear fractures mechanics considering the size effect; Kim's equation (Kim, Park 1996), based on basic shear transfer mechanisms, a modified Bazant's size effect law and test data; Collins' equation (Collins, Kuchma 1999), resulting from an enhancement of the modified compression field theory based on a hypothesis that crack spacing causes size effect; Rebeiz's equation (Rebeiz 1999), obtained from multiple regression analysis; and Khuntia's equation (Khuntia, Stojadinovic 2001), based on basic principles of mechanics and parametric study of experimental data. All the equations considered within the scope of this study are summarized in Table 1. These equations are applied to a database consisting of 282 specimens so that the results of the equations can be compared to the test results.

Based on the test results, Jelic *et al.* (1999) reported that dowel action cannot be considered as a viable component in the shear mechanism of a cracked reinforced concrete beam section without stirrups. The main factor resisting the applied shear force is the shear resistance of the uncracked concrete. According to Zararis and Papadakis (2001), and Kotsovos and Pavlovic (1998) the compression zone of intact concrete prevents shear-slip of the crack surfaces. Reinhardt and Walraven (1982) reported that the tension zone damaged by the flexural cracks does not significantly contribute to the shear resistance of the beams. Normally, dowel action is not very significant in members without stirrups, since the maximum shear in a dowel is limited by the tensile strength of the cover concrete supporting the dowel (Bauman, Rüsch 1970). Therefore, the aggregate interlock along the crack surfaces and the dowel action of longitudinal reinforcement do not significantly contribute to the shear strength of the beams (Kotsovos, Pavlovic 1998). According to Jelic *et al*. (1999), the dowel action of longitudinal reinforcement placed in one layer can be neglected for RC beams without stirrups. In the present study, dowel action is neglected for simplicity and conservatism.

3. The prediction of diagonal cracking strength of beams without stirrups

The shear failure of reinforced concrete members without stirrups initiates in the form of diagonal cracks, which later propagates through the beam web, when the principal tensile stress within the shear span exceeds the tensile strength of concrete. For a RC slender beam where a/d is greater than 2.5, the shear strength at section is primarily concerned with the effective shear depth of critical diagonal crack and the tensile strength of concrete. The effective shear depth, based on Khuntia's equation (Khuntia, Stojadinovic 2001), is obtained using the depth of neutral axis and the compressive strain in concrete. The diagonal cracking strength equation can be obtained by using a number of simplifying assumptions, and is based on the basic principles of mechanics, but cracking included, and the theory of elasticity. The proposed equation is compared with the test results of high-strength concrete (HSC) beams with compressive strength of $f_c \geq 55$ MPa, and normal-strength concrete (NSC) beams with lower f_c values reported in the literature.

In 1902 Mörsch derived the shear stress distribution for a RC beam containing flexural cracks. Mörsch predicted that shear stress would reach its maximum value at the neutral axis and would then remain constant from the neutral axis down to the flexural steel (Cladera 2003). The value of this maximum shear stress would be:

$$\tau_o = V_o / \left(b_w z\right), \tag{1}$$

where: b_w is the web width; z is the flexural lever arm.

According to Zink (2000), the stress distribution for τ_o shown in Fig. 1a is neither able to describe the shear cracking force in quality nor in quantity. New approaches (Khuntia, Stojadinovic 2001; Zink 2000) were proposed to adopt the influence of the components in the cracked flexural tension zone more accurately. Khuntia and Stojadinovic (2001) modeled the shear stress distribution as a parabola with an integral factor of 2/3 over the effective shear depth with the maximum value at the neutral axis (Fig. 1d). The shear capacity of beams without stirrups, V_o, carried in the uncracked compressive zone, is determined by integrating the shear stress and is expressed as follows:

$$V_o = \frac{2}{3} b_w c_1 f_t, \tag{2}$$

where c_1 is the effective shear depth and f_t is the tensile strength of concrete.

The shear capacity controlled by tension is normally less than that controlled by compression, regardless of the magnitude of flexural deformation. This is because the tensile strength of concrete is much less than the compressive strength (Park *et al.* 2006). According to Khuntia

Table 1. Existing shear strength models for slender beams without stirrups

Investigator	Shear strength models
ACI318 (2008)	$v_c = 0.16\sqrt{f_c} + 17\rho\left(V_u d / M_u\right) \le 0.29\sqrt{f_c}$ or $v_c = 0.17\sqrt{f_c}$, f_c in MPa
TS500 (2000)	$v_c = \left(0.2275\sqrt{f_c}\right)$, f_c in MPa
CSA (1994)	$v_c = 0.2\sqrt{f_c}$, f_c in MPa, ($d \le 300$ mm), $v_c = \left(\frac{260}{1000+d}\right)\sqrt{f_c} \ge 0.1\sqrt{f_c}$, f_c in MPa, and d in mm(d>300 mm)
NZS (1995)	$v_c = (0.07 + 10\rho)\sqrt{f_c}$, f_c in MPa, $a/d \ge 2$
EN 1992-1-1: 2004 (2004)	$v_{rd,c} = 0.18k\left(100\rho f_c\right)^{1/3} \ge 0.035k^{3/2}\sqrt{f_c}$, f_c in MPa, $k = 1 + \sqrt{\frac{200}{d}} \le 2.0$, $\rho = \frac{A}{b_w d} \le 0.02$, d in mm
CEB-FIP90 (1993)	$v_{cr} = 0.15\left(1+\sqrt{\frac{200}{d}}\right)\left(100\rho f_c\right)^{1/3}\left(\frac{3}{a/d}\right)^{1/3}$, f_c in MPa, and d in mm
Zsutty (1971)	$v_u = 2.2\left(f_c \rho \frac{d}{a}\right)^{1/3}$, f_c in MPa, $\left(a/d\right) \ge 2.5$
Okamura and Higai (1980)	$v_c = 0.2\frac{\left(100\rho f_c\right)^{1/3}}{d^{1/4}}\left(0.75 + \frac{1.40}{a/d}\right)$, f_c in MPa, and d in m
Bazant and Kim (1984)	$v_u = 0.54\sqrt[3]{\rho}\left(\sqrt{f_c} + 249\sqrt{\frac{\rho}{(a/d)^5}}\right)\left(\frac{1+\sqrt{5.08/d_a}}{\sqrt{1+d/(25d_a)}}\right)$, f_c in MPa, d in mm
Kim and Park (1996)	$v_u = 3.5 f_c^{\alpha/3}\rho^{3/8}\left(0.4 + \frac{d}{a}\right)\left(\frac{1}{\sqrt{1+0.008d}} + 0.18\right)$, f_c in MPa, and d in mm, $\alpha = 2 - (a/d)/3$ for $1.0 \le \frac{a}{d} < 3.0$, $\alpha = 1$ for $\frac{a}{d} \ge 3.0$
Collins and Kuchma (1999)	$v_c = \frac{245}{1275 + \left(\frac{25S_X}{\left(d_a + 16\right)}\right)}\sqrt{f_c}$ $S_X \approx 0.9d$, f_c in MPa, d and d_a in mm
Rebeiz (1999)	$v_c = 0.4 + \sqrt{f_c \rho \frac{d}{a}\left(2.7 - 0.4A_d\right)}$, f_c in MPa $A_d = a/d$ for $\left(a/d\right) < 2.5$ and $A_d = 2.5$ for $\left(a/d\right) \ge 2.5$.
Khuntia and Stojadinovic (2001)	$v_c = 0.54\sqrt[3]{\rho\left(f_c \frac{V_c d}{M_u}\right)^{0.5}}$, f_c in MPa, $\frac{M_u}{V_c d} = \frac{a}{d} - 1$

Note: v_c – the shear strength of RC members without stirrups; v_{cr} – diagonal cracking shear strength; v_u – ultimate shear strength. Magnitudes of v_c and v_u are considered to be equal to v_{cr} in calculating shear strength.

and Stojadinovic (2001), the shear stress distribution is modeled as parabolic over the effective shear depth with the maximum value at the neutral axis such that the maximum shear stress over the effective cross section equals to $\tau_o = V_o / (2/3 b_w c_1)$, where b_w is the width of section and c_1 is the effective shear depth. The shear failure of RC members without stirrups initiate when the principal tensile stress within the shear span exceeds the tensile strength of concrete and a diagonal crack propagates through the beam web. Mathematically:

$$\tau_o = \frac{V_o}{\frac{2}{3}b_w c_1} = f_t, \quad (3)$$

where f_t is the tensile strength of concrete and

$$V_o = \frac{2}{3} f_t b_w c_1 = v_o b_w d \quad (4)$$

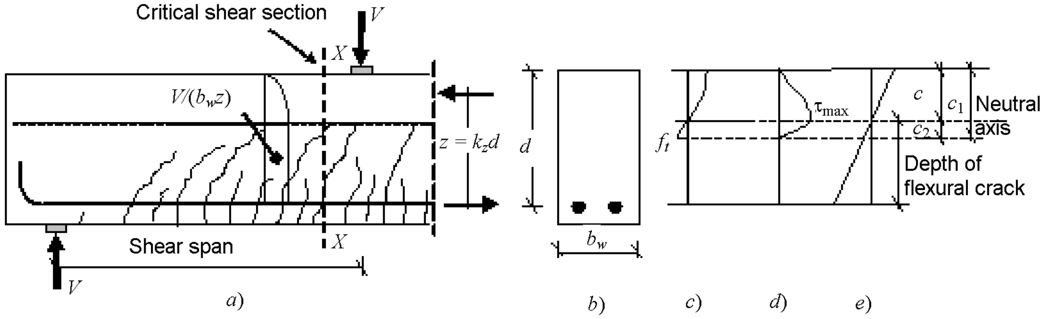

Fig. 1. Shear stress and strain distribution in a RC beam with flexural cracks: a) typical crack pattern and shear stress distribution (adapted from Cladera (2003)); b) cross-section X-X; c) distribution of concrete stresses (Khuntia, Stojadinovic 2001); d) shear stress distribution (Khuntia, Stojadinovic 2001); e) longitudinal strain distribution

is the shear force. The procedure proposed by Khuntia and Stojadinovic (2001) for the shear strength of RC members without stirrups, at the design section under the effect of the factored bending moment M_u and axial load P_u, calculates the effective shear depth c_1, using the method of satisfaction of strain compatibility and equilibrium conditions:

$$c_1 = c\left(1 + \frac{\varepsilon_{cr}}{\varepsilon_c}\right), \quad (5)$$

where c is the depth of neutral axis, ε_c is the compressive strain in concrete, which is taken as 0.002, and ε_{cr} is the cracking strain value in concrete, which is taken as the ratio of the tensile strength of concrete f_t to its modulus of elasticity E_c:

$$\varepsilon_{cr} = \frac{f_t}{E_c}. \quad (6)$$

Since concrete is relatively weak and brittle in tension, cracking is expected when significant tensile stress is induced in a RC member (ACI224 1992). The tensile strength of plain concrete f_t, ranges from about 0.25 to $0.50\sqrt{f_c}$ (Nilson, Darwin 1997; Paulay, Priestley 1992; Carreira, Chu 1986). The tensile strength f_t of concrete has been taken as $0.3 f_c^{2/3}$ (EN 1992-1-1:2004 2004; CST 49 1968). In this study, the direct tensile strength is assumed as $f_t = 0.3 f_c^{2/3}$ and the modulus of elasticity E_c is taken as $4700\sqrt{f_c}$ (ACI318 2008) for all levels of concrete strength.

Substituting Eq. (5) and Eq. (6) into Eq. (4), the principal shear strength becomes:

$$v_o = 0.2 f_c^{2/3}\left(\frac{c}{d}\right)\left(1 + 0.032 f_c^{1/6}\right), \quad (7)$$

where f_c is the compressive strength of concrete in MPa and c/d is the ratio of neutral axis depth to effective depth, which is the positive root of the second order equation given by Eq. (8). At any section of the member, the depth of the flexural crack is expected to increase with bending moment. Thus, the diagonal cracking strength would decrease with increasing bending moment (Khuntia, Stojadinovic 2001). The strain of the tensile reinforcement can be taken as $\varepsilon_s = \varepsilon_c (d - c)/c$, where ε_c is the concrete strain at extreme fiber at maximum stress, 0.002. Using the equality of the concrete compression force, $F_{cc}^{'} = (2/3) f_c b_w c$, and the normal steel force, $F_s = \rho b_w d E_s \varepsilon_c (d - c)/c$, and considering that $E_s = 2.10^5$ MPa, the depth of the neutral axis has been previously proposed as (Zararis, Papadakis 2001):

$$\left(\frac{c}{d}\right)^2 + 600\frac{\rho}{f_c}\frac{c}{d} - 600\frac{\rho}{f_c} = 0. \quad (8)$$

Eq. (7) does not capture the effects of slenderness and size on the diagonal cracking strength, which were considered by many researchers and codes. The diagonal cracking strength of RC beams decreases with increasing member depth and slenderness. In order to take into account the effects of slenderness and size, Eq. (7) is calibrated to the test results available in the literature.

4. Calibration by comparison with the effect of slenderness and size

In order to obtain more accurate diagonal cracking strength, the principal shear strength equation is modified by the factors of the effective depth and slenderness ratio (a/d). Based on the principal shear strength v_o carried by the compression zone, the influence of the slenderness and size can be considered with the factors $k(a/d)$ and $k(1/d)$. The diagonal cracking strength causing shear tension failure can be written as follows:

$$v_c = v_o k(a/d) k(1/d), \quad (9)$$

where $k(a/d)$ and $k(1/d)$ are the coefficient of slenderness and size effect, respectively.

4.1. Slenderness effect on diagonal cracking strength

The weak influence of a/d is neglected in some design formulas (ACI318 2008; TS500 2000; CSA 1994; NZS 1995; EN 1992-1-1:2004 2004; Collins, Kuchma 1999;

Arslan 2005, 2008). Tension stiffening causes a minor influence of *a*/*d*, which is described with a coefficient $k(a/d)$ in some other design formulas (CEB-FIP90 1993; Zsutty 1971; Okamura, Higai 1980; Bazant, Kim 1984; Kim, Park 1996; Rebeiz 1999; Khuntia, Stojadinovic 2001). CEB-FIP90 equation is proposed to determine the relationship between *a*/*d* and the diagonal cracking strength, leading to the following expression for slender beams: $v_c \propto \left(3/\left(a/d\right)\right)^{1/3}$. For a slenderness ratio $\left(a/d\right)=3.0$, a complete crack formation can be observed when shear failure occurs. Therefore, the coefficient of slenderness ratio is set to 1.0 for $\left(a/d\right)=3.0$. The exponent in the formula has a relatively small effect on the coefficient of slenderness ratio. The cracking shear strength decreases 21% even *a*/*d* is increased to 6.0 from 3.0. According to the Okamura and Higai (1980), the relationship between *a*/*d* and the cracking shear strength is set to $v_c \propto \left(0.75+1.40/\left(a/d\right)\right)$. The coefficient of slenderness ratio is set to 1.0 for $\left(a/d\right)=5.6$. Zsutty's equation (Zsutty 1971) shows that v_c is proportional to $\left(1/\left(a/d\right)\right)^{1/3}$. According to Kim and Park (1996), the relationship between *a*/*d* and the cracking shear strength is set to $v_c \propto \left(0.4+1/\left(a/d\right)\right)$ for slender beams. The coefficient of slenderness ratio is set to 1.0 for $\left(a/d\right)=1.7$.

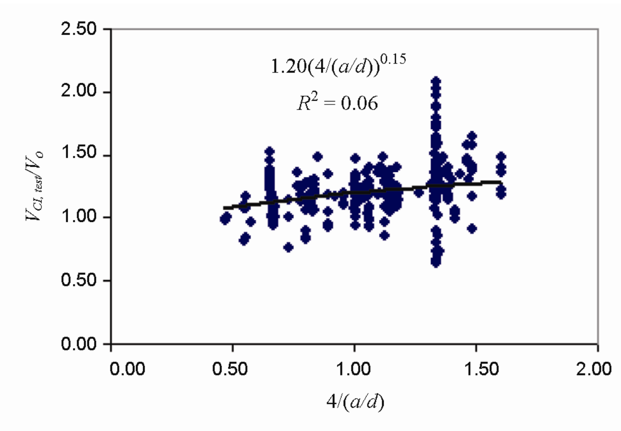

Fig. 2. Effect of slenderness ratio on cracking shear strength

A significant influence of tension stiffening can only be expected for values of *a*/*d* smaller than 4. For ratios of *a*/*d* higher than 4 there is no significant influence of tension stiffening because of the already completed crack formation in the critical area of the shear span (Zink 2000). In this study, a regression analysis is undertaken to identify the influence of $4/\left(a/d\right)$ on the diagonal cracking strength of RC slender beams without stirrups using the results of existing 282 experimental data. The variation of the ratio of experimental cracking shear strength ($v_{c,test}$) to the principal shear strength v_o of slender beams can be expressed as follows:

$$\frac{v_{c,test}}{v_o}=\left(\frac{4}{a/d}\right)^{0.15}. \quad (10)$$

The variation of $v_{c,test}/v_o$ with the varying level of $\left(4/(a/d)\right)^{0.15}$ is illustrated in Fig. 2. A new design expression is proposed for the diagonal cracking strength based on the principal shear strength for slender beams by considering the slenderness effect.

4.2. Size effect on diagonal cracking strength

The influence of the effective depth *d* is neglected in some design formulas (ACI318 2008; TS500 2000; NZS 1995; Zsutty 1971; Rebeiz 1999). However, in generally, size effect on the cracking shear strength is significant and is described with a coefficient *k*(1/*d*) in some other design formulas (CSA 1994; EN 1992-1-1:2004 2004; CEB-FIP90 1993; BS 8110 1997; Okamura, Higai 1980; Bazant, Kim 1984; Kim, Park 1996; Collins, Kuchma 1999). The CSA Code (1994) includes a term to account for the size effect in its simplified shear design expression but does not take the reinforcing steel ratio, ρ, into account. This shows the concern of this code regarding the size effect phenomenon. When the effective depth *d* is quite large, the CSA Code (1994) equation considers an over strong asymptotic size effect $v_c \propto d^{-1}$, which is contrary to the point of view that the linear elastic fracture mechanics size effect $v_c \propto d^{-1/2}$ for very large beam depths (Bazant, Yu 2005).

BS 8110 (1997) equation relates the cracking shear strength to the size effect as follows:

$$v_c \propto \left(\frac{400}{d}\right)^{0.25}. \quad (11)$$

Based on the regression analysis, v_c is taken as proportional to $\left(400/d\right)^{0.25}$ in this study to identify the size effect, which is similar to BS 8110 equation.

4.3. Proposed diagonal cracking strength equation for RC slender beams without stirrups

Based on the principal shear strength v_o carried in the compression zone, considering the influence of parameters; the slenderness ratio (*a*/*d*) and size effect (1/*d*), the diagonal cracking strength of RC slender beams without stirrups can be expressed as follows:

$$v_c = 0.2 f_c^{2/3}\left(\frac{c}{d}\right)\left(1+0.032 f_c^{1/6}\right)\left(\frac{4}{\left(a/d\right)}\right)^{0.15}\left(\frac{400}{d}\right)^{0.25}. \quad (12)$$

Eq. (12) is proposed for RC beams with a shear span to depth ratio $\left(a/d\right)$ equal to or greater than 2.5.

5. Evaluation of proposed equation

The diagonal cracking strength equation was applied to the 282 specimens that had been tested by 22 researchers.

These specimens were subjected to single- or two-point loads at mid-span. The specimens have a broad range of design parameters: $0.47 \leq \rho \leq 5.01$ (%), $2.50 \leq a/d \leq 8.52$, $6.1 \leq f_c \leq 53.9$ MPa and $41 \leq d \leq 483$ mm for NSC and $0.33 \leq \rho \leq 6.64$ (%), $2.50 \leq a/d \leq 6.00$, $56.5 \leq f_c \leq 91.8$ MPa, and $184 \leq d \leq 822$ mm for HSC. The effects of concrete compressive strength, slenderness ratio and flexural reinforcement ratio on the proposed diagonal cracking strength of RC slender beams without stirrups are discussed below.

Fig. 3 compares the proposed diagonal cracking strength obtained from Eq. (12) with experimental results for NSC beams (Taylor 1960; Bazant, Kazemi 1991; Mphonde, Frantz 1984; Moody *et al.* 1954; Diaz de Cossio, Siess 1960; Van den Berg 1962; Taylor, Brewer 1963; Bresler, Scordelis 1963; Mathey, Watstein 1963; Mattock 1969; Krefeld, Thurston 1966; Cho 2003; Cladera, Mari 2005) and HSC beams (Mphonde, Frantz 1984; Van den Berg 1962; Cho 2003; Cladera, Mari 2005; Ahmad *et al.* 1986; Elzanaty *et al.* 1986; Kwak *et al.* 2002; Shah, Ahmad 2007; Sneed, Ramirez 2010). The mean values (MV) and the standard deviations (SD) of the ratio of the experimental cracking shear strength to the proposed diagonal cracking strength are 1.108 and 0.118 for NSC, 1.012 and 0.153 for HSC, respectively.

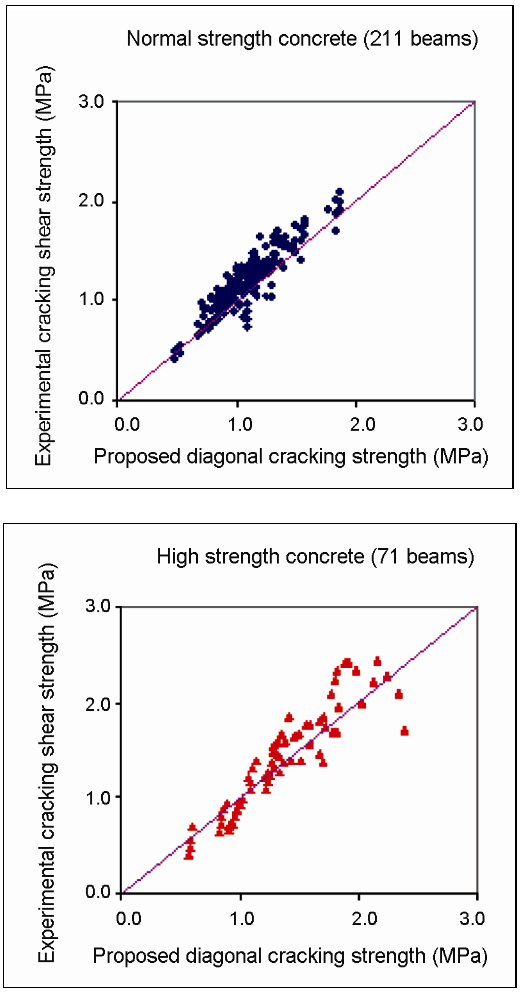

Fig. 3. Proposed diagonal cracking strength values using Eq. (12) versus experimental cracking shear strength values

Figs 4–6 show the errors which can be induced by the discrepancies of a/d, ρ and f_c. The ratio of the experimental to the proposed shear strength is not significantly influenced by increasing a/d, ρ and f_c. However, experimental data are not homogeneous as shown in Figs 4–6.

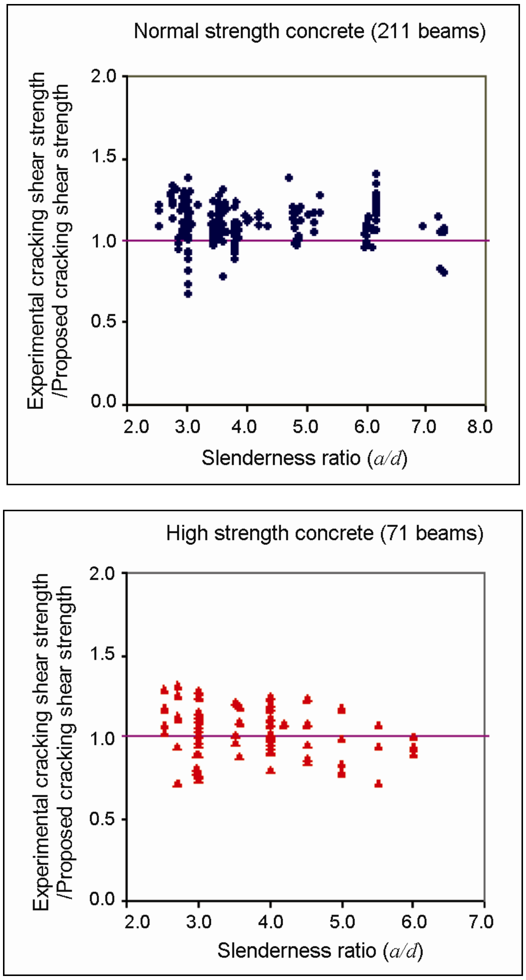

Fig. 4. Comparing experimental cracking shear strength values with proposed diagonal cracking strength of Eq. (12) for various slenderness ratios

According to Leonhardt and Walter (1962), if the flexural reinforcement ratio is kept constant, the mode of failure of rectangular RC beams without stirrups depends on the slenderness ratio. If the reinforcement ratio is greater than approximately 1.8%, shear failure is more critical than flexural failure for slenderness ratios between 1 and 7. Based on test results, ACI Building Code provisions are nonconservative for low values of the flexural reinforcement ratio, ρ, and, therefore, unsafe for beams without stirrups (Ahmad *et al.* 1986). According to TS500 (2000), the amount of the flexural reinforcement ratio is limited within the range of $\rho \leq 2.0\%$. However; the strength of members with low reinforcing ratios was rarely investigated in the past and is often overestimated in the present codes (ASCE-ACI445 1999). As shown in Fig. 5, the test results of the cracking shear strength of HSC slender beams with low reinforcing ratios are very limited ($\rho < 1.0\%$), consequently further research is required to verify the proposed equations for HSC beams.

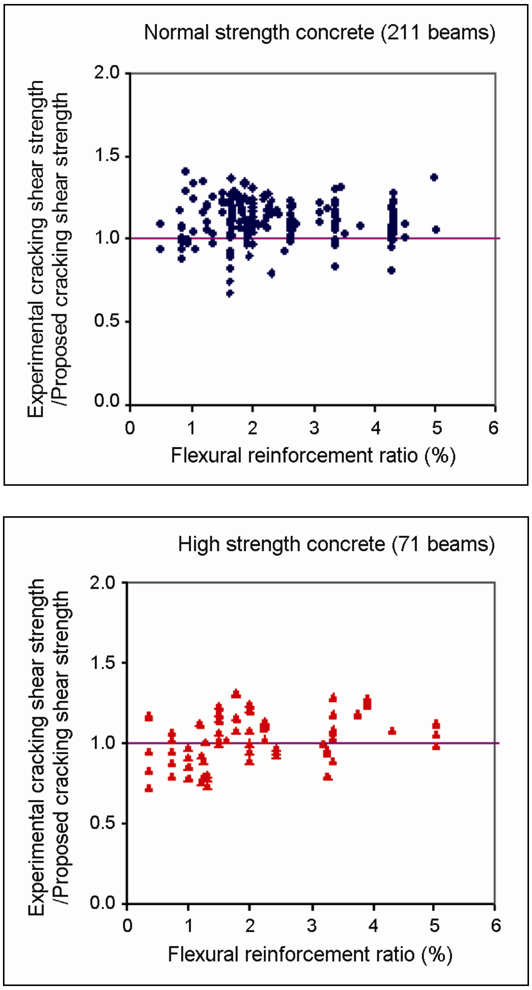

Fig. 5. Comparing experimental cracking shear strength values with proposed diagonal cracking strength of Eq. (12) for various flexural reinforcement ratios

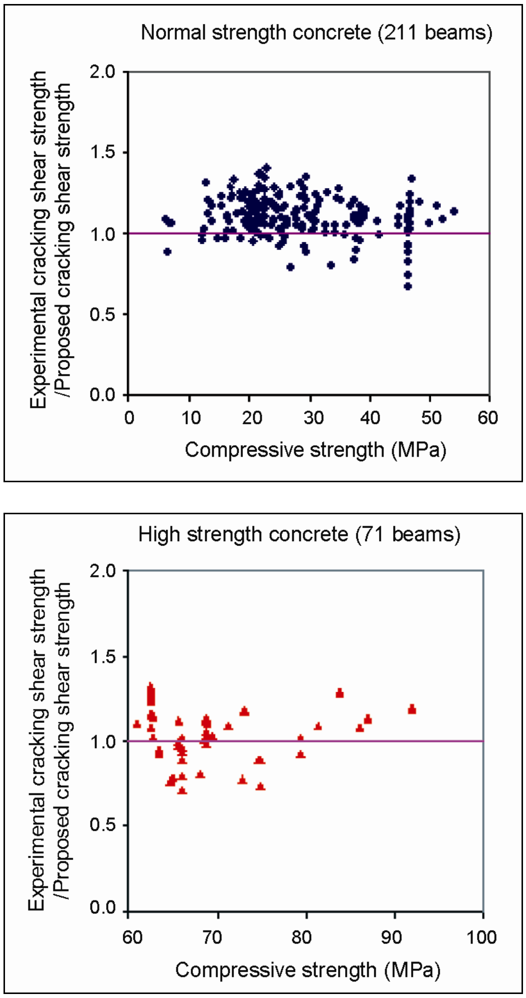

Fig. 6. Comparing experimental cracking shear strength values with proposed diagonal cracking strength of Eq. (12) for various compressive strength values

Fig. 6 shows the ratio of the experimental to the proposed diagonal cracking strength for various values of concrete compressive strength. The ratio of the experimental to the proposed diagonal cracking strength is not influenced significantly by f_c for both NSC and HSC. Since the test data for HSC members are very limited, further research is required to verify the proposed equations for HSC beams.

Table 2 summarizes the comparisons of the predictions obtained from the proposed equation, ACI318 Building Code (2008), TS500 (2000), CSA Code (1994), NZS (1995), EN 1992-1-1:2004 (2004), CEB-FIP90 (1993), Zsutty's equation (Zsutty 1971), Okamura's equation (Okamura, Higai 1980), Bazant's equation (Bazant, Kim 1984), Kim's equation (Kim, Park 1996), Collins' equation (Collins, Kuchma 1999), Rebeiz's equation (Rebeiz 1999), and Khuntia's equation (Khuntia, Stojadinovic 2001) with the test results available in the literature. The resulting coefficient of variation (COV) of the ratio of the experimental value (NSC) to the prediction from the proposed equation is 27% of those obtained for CSA Code prediction, 37% of those obtained for NZS Code prediction, 52% of those obtained for Bazant's equation, 58% of those obtained for ACI318 and TS500 predictions, 59% of those obtained for Collins' and Khuntia's equations, 66% of those obtained for Rebeiz's and Zsutty's equations, 74% of those obtained for EN prediction, 80% of that obtained for Kim's equation, 83% of that obtained for CEB-FIP prediction and 88% of that obtained for Okamura's equation.

The resulting COV of the ratio of the experimental value (NSC and HSC) to the prediction from the proposed equation is 29% of those obtained for CSA Code prediction, 42% of those obtained for NZS Code prediction, 49% of those obtained for ACI318 and TS500 Code predictions, 51% of those obtained for Collins' equation, 62% of those obtained for Bazant's equation, 68% of those obtained for Rebeiz's equation, 69% of those obtained for Zsutty's equation, 75% of those obtained for EN prediction, 81% of that obtained for Kim's equation, 83% of that obtained for CEB-FIP prediction and 88% of that obtained for Okamura's equation.

Table 2. Verification of proposed Eq. (12) using entire database

	$f_c \leq 55$ MPa (NSC)			$f_c > 55$ (HSC)			NSC & HSC		
	MV	SD	COV	MV	SD	COV	MV	SD	COV
Exp. / Prop.	1.108	0.118	0.106	1.012	0.153	0.151	1.084	0.134	0.123
Exp./ ACI318	1.345	0.246	0.183	1.011	0.366	0.362	1.261	0.316	0.250
Exp. / TS500	1.005	0.184	0.183	0.755	0.273	0.362	0.942	0.236	0.250
Exp. / CSA94	1.423	0.553	0.388	0.959	0.368	0.383	1.323	0.565	0.427
Exp. / NZS	0.771	0.219	0.284	0.615	0.148	0.240	0.731	0.214	0.293
Exp./ EN92	1.333	0.190	0.143	1.178	0.231	0.196	1.294	0.212	0.164
Exp./ CEB-FIP	1.148	0.147	0.128	1.006	0.168	0.167	1.113	0.165	0.148
Exp./ Zsutty's	1.001	0.160	0.160	0.875	0.173	0.197	0.970	0.172	0.177
Exp./ Okamura's	0.948	0.114	0.120	0.833	0.135	0.162	0.919	0.129	0.141
Exp./ Bazant's	0.770	0.156	0.203	0.703	0.121	0.172	0.753	0.151	0.200
Exp. / Kim's	0.931	0.123	0.132	0.802	0.130	0.162	0.898	0.137	0.153
Exp. / Collins'	1.344	0.241	0.179	1.035	0.354	0.342	1.267	0.304	0.240
Exp. / Rebeiz's	1.113	0.179	0.161	0.959	0.192	0.201	1.074	0.194	0.181
Exp. / Khuntia's	1.146	0.205	0.179	1.159	0.234	0.202	1.149	0.212	0.185

Exp. = Experimental Prop. = Proposed

6. Conclusions

On the basis of results obtained in this study, the following conclusions are drawn:

1. It can be seen that the proposed diagonal cracking strength equation (Eq. (12) for RC slender beams results in the lowest coefficient of variation (COV) for the ratio of experimental value to the predicted value for NSC and HSC beams. Hence Eq. (12) provides better results than six codes of practice and seven equations proposed by different researchers for the prediction of diagonal cracking strength of RC beams with NSC and HSC. However, further research is required to verify the proposed equation since the test data for HSC members is very limited.

2. The mean value (MV) of the experimental cracking shear strength to the proposed diagonal cracking strength is 1.108 for NSC and 1.012 for HSC beams. Therefore, it can explain that the contribution of dowel action to the diagonal cracking strength provides additional conservation.

3. The predictions by the proposed equation for the shear strength of test beams are relatively better, whereas ACI318, CSA, EN 1992-1-1:2004, Collins' equation is excessively conservative for most of the test results and the NZS and Bazant's equations give unsafe results for slender beams.

4. The ratio of the experimental to the proposed diagonal cracking strength is not significantly influenced by increasing a/d, ρ and f_c, but it is important to note that test data are not homogeneous.

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SKERSINE ARMATŪRA NEARMUOTŲ GELŽBETONINIŲ SIJŲ TEMPIAMASIS SUIRIMAS ĮSTRIŽAJAME PJŪVYJE

G. Arslan

S a n t r a u k a

Gelžbetoninių sijų suirimas įstrižajame pjūvyje – viena pagrindinių problemų statybos inžinerijoje. Tačiau skersine armatūra nearmuotų gelžbetoninių sijų įstrižasis tempiamasis stipris nėra visiškai ištirtas. Šiame straipsnyje nagrinėjamas siaurų, be skersinės armatūros gelžbetoninių sijų įstrižojo pjūvio pleišėjimas. Siaurose sijose plyšiai tempiamojoje zonoje atsiranda anksčiau negu įstrižajame pjūvyje. Taikant klasikinius mechanikos principus ir tamprumo teoriją, pasiūlyta normalaus stiprio arba stipriojo betono sijų įstrižojo pjūvio atsparumo pleišėjimui apskaičiavimo lygtis. Siūloma lygtis, pagrįsta šešių projektavimo normų reikalavimais ir septyniomis kitų autorių lygtimis bei palyginta su literatūroje pateiktais 282 sijų eksperimentinių tyrimų rezultatais. Nustatyta, kad pagal siūlomą lygtį atlikti skaičiavimai gerai sutampa su eksperimentiniais rezultatais.

Reikšminiai žodžiai: gniuždomasis stipris, gelžbetonis, supleišėjimas, įstrižojo pjūvio stipris, liauna sija, įstrižasis tempimas.

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