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EVALUATION OF STRENGTH OF SPHERICAL CONTACT PAIRS OF AVIATION STRUCTURES

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Abstract. The strength criterion is used to evaluate the spherical contact pairs of the aviation structures. The strength criterion helps evaluate triaxial tension in the contact pair. Calculations reveal the acceptability of the suggested criterion.

Keywords: aviation construction, contact pair, strength criterion.

1. Introduction

Contact pairs play an important role in aviation structures when the reliability and durability of an aircraft is being determined (Filonenko *et al*. 2008, Filonenko *et al*. 2009). The materials used in contact pairs are selected according to many reliability factors. The better the contact pair materials are, the better, more reliable, and more durable the work of the links and contact parts are. Before the selection of materials for contact, it is necessary to evaluate the conditions under which the structure is going to work and what static and dynamic loads will be formed during work. When these conditions are known, the materials can be selected for certain work conditions. When such calculations are done, it is important to determine the threshold states of stress and deformation, i.e. to evaluate contact stress for triaxial load. Hertz has analysed this contact problem the most thoroughly (Johnson 1985). He analysed a solid that cannot be deformed and that affects an elastic area where external and internal stresses are formed, which affects the destruction of materials. Efforts were later put into

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evaluating more complex work conditions by introducing correction coefficients (Morozov *et al.* 1999). This worktries to apply a strength criterion for complex stresses with regard to the material's behaviour during triaxial load.

2. Evaluation of threshold states of stresses and deformation in contact pairs

When the real bodies contact, various states of stress and deformation may appear. All the contact problems may be divided into two groups:

1. The stress fields and deformations do not have big gradients. The solutions do not require complex strength criteria.
2. The stress fields have big gradient in various directions. When such pairs of attrition are projected, it is necessary to evaluate the level of gradients and to apply complex strength criteria.

The stress field in the contact zone under the impact of accumulated force was described for the first time by Boussinesq (Fig 1) (Žiliukas 2006).

The types of stress are described as follows:

$$\begin{aligned}
\sigma_r &= \frac{F}{\pi r_0^2}\left\{\left(\frac{1-2\nu}{4}\right)\sec^2\frac{\phi}{2}-\frac{3}{2}\cos\phi\sin^2\phi,\right.\\
\sigma_z &= -\frac{F}{\pi r_0^2}\left(\frac{3}{2}\cos^2\phi\right),\\
\sigma_\theta &= \frac{F}{\pi r_0^2}\left\{\left(\frac{1-2\upsilon}{4}\right)\left(\cos\phi-\frac{1}{2}\sec^2\frac{\phi}{2}\right)\right\},\\
\tau_{rz} &= -\frac{F}{\pi r_0^2}\left(\frac{3}{2}\cos^2\phi\sin\phi\right).
\end{aligned} \tag{1}$$

σ_r, σ_z, and σ_θ are radial, axial and circular stresses respectively; F is accumulated force; r_0 is distance from the analyzed point to the adding point of force; ν is Poisson's ratio; ϕ is the angle between the acting line of force F until a straight line drawn from the beginning of the coordinates on the adding point of force until the point analysed (Fig 1).

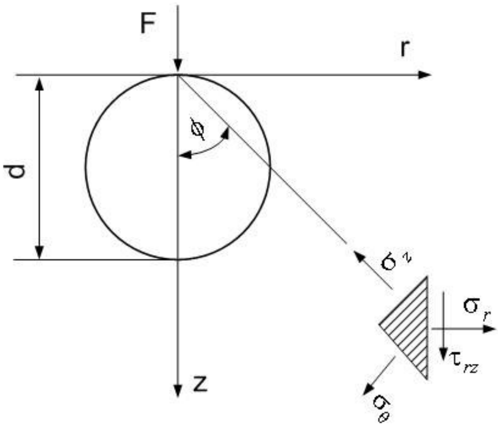

Fig 1. Stresses field in the contact zone

The main types of stress according to Boussinesq equation are written as follows:

$$\begin{aligned}
\sigma_1 &= \sigma_r\sin^2\phi+\sigma_z\cos^2\phi-2\tau_{rz}\sin\phi\cos\phi,\\
\sigma_2 &= \sigma_\theta,\\
\sigma_3 &= \sigma_r\cos^2\phi+\sigma_z\sin^2\phi+2\tau_{rz}\sin\phi\cos\phi.
\end{aligned} \tag{2}$$

The angle ϕ is calculated according to the formula:

$$tg2\phi=\frac{2\tau_{rz}}{\sigma_z-\sigma_r}. \tag{3}$$

A. N. Dinik analyzed the Hertz contact equation for a circular contact area in the direction of axis z (Fig 2) and received the following (Morozov *et al.* 1999):

$$\begin{aligned}
\sigma_{33} &= -\frac{p_0}{1+(z/\theta)^2},\\
\sigma_{22} &= \sigma_{11} = -p_0\left[(1+\nu)\left(1-\frac{z}{\theta}arctg\frac{\theta}{z}\right)-\frac{0.5}{1+(z/\theta)^2}\right],\\
\sigma_{12} &= \sigma_{23} = \sigma_{31} = 0.
\end{aligned}$$

ρ_0 stands for spherically contacting bodies; when the resilience modules of materials are $E_1 = E_2 = E$, Poisson's ratios are $\nu_1=\nu_2=\nu=0.3$, is calculated in the following way (Žiliukas 2006):

$$p_0 = 0.5784\sqrt[3]{\frac{F}{\beta_0^2}\left(\frac{R_1 \pm R_2}{R_1R_2}\right)}. \tag{5}$$

β_0 is coefficient of material properties calculated as follows:

$$\beta_0=\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}.$$

R_1 is sphere's radius. R_2 is ball's radius. F is load.

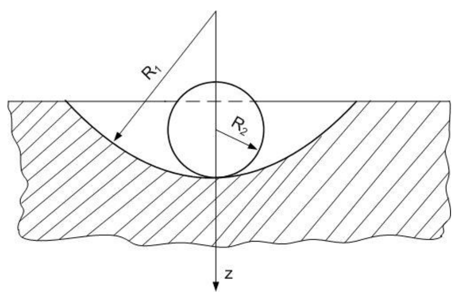

Fig 2. Contact of spherical surfaces

In the formula (5) the sign "-" is used when the sphere is concave and "+" when it is convex. In this case, it will be "-" in figure 2.

The radius of the contact surface area θ is calculated as follows (Žiliukas 2006):

$$\theta = 0.9086\sqrt[3]{p_0 F \frac{R_1 R_2}{R_1 \pm R_2}}. \quad (6)$$

Formula (6) uses the sign "-" of figure 2.

The biggest tangential stresses (3)

$$\tau_{\max} = \frac{p_0}{2}\left[\frac{1.5}{1+(z/a)^2} - (1+\nu)\left(1 - \frac{z}{a} arctg \frac{a}{z}\right)\right]. \quad (7)$$

τ_{max} stress appears at the point z = 0.48α and is equal to $\tau_{max} = 0.310\rho_0$ at υ = 0.3.

τ_{max} stress cannot determine strength with regard to especially strong materials, however, because τ_{max} affects just the plasticity of plastic materials. Thus it would be best to evaluate the retention capacity of contact pairs using the universal strength criterion that is used for semi-brittle materials.

3. Results

When the radius of a sphere is R_1=80 mm and the radius of a ball R_2=10 mm in formulas (5) and (6) at $\upsilon_1 = \upsilon_2 = \upsilon = 0.3$ and $E_1 = E_2 = E = 2\cdot10^5$ MPa, we receive:

$$\frac{p_0}{a} = 2.8\cdot10^5 (\frac{MN}{m^3}).$$

Let us analyze the cases when the radius of the contact surface area α has the values of 5 mm, 2.5 mm and 10 mm.

Thus when α=5 mm, ρ_0=1400 MPa.

And when α=1.0 mm, ρ_0=280 MPa.

We will use formulas (4) to calculate the principal stress when *z* changes from 0 to α.

Table 1 presents the calculation results when α=5 mm:

Table 1. Principal stress

z, mm	*z* / α	A/ *z*	σ_{33}, MPa	$\sigma_{22} = \sigma_{11}$, MPa
0	0	∞	-1400	-1120
1	0.2	5	-1346	-644
2	0.4	2.5	-1207	-350
3	0.6	1.66	-1029	-164
4	0.8	1.25	-854	-90
5	1.0	1.0	-700	-41

Table 2 presents the calculation results when α=2.5 mm:

Table 2. Principal stress

z, mm	*z* / α	α/ *z*	σ_{33}, MPa	$\sigma_{22} = \sigma_{11}$, MPa
0	0	∞	-700	-560
0.5	0.2	5	-673	-322
1	0.4	2.5	-603	-175
1.5	0.6	1.66	-514	-82
2	0.8	1.25	-427	-45
2.5	1.0	1.0	-350	-20

Table 3 presents the calculation results when α=1.25 mm:

Table 3. Principal stress

z, mm	*z* / α	α/ *z*	σ_{33}, MPa	$\sigma_{22} = \sigma_{11}$, MPa
0	0	∞	-350	-280
0.25	0.2	5	-336	-161
0.5	0.4	2.5	-301	-87
0.75	0.6	1.66	-257	-41
1	0.8	1.25	-213	-22.5
1.25	1.0	1.0	-175	-10

Table 4 presents the calculation results when α=1.0 mm:

Table 4. Principal stress

z, mm	*z* / α	α/ *z*	σ_{33}, MPa	$\sigma_{22} = \sigma_{11}$, MPa
0	0	∞	-280	-224
0.2	0.2	5	-269	-129
0.4	0.4	2.5	-241	-70
0.6	0.6	1.66	-206	-32.8
0.8	0.8	1.25	-171	-18
1.0	1.0	1.0	-140	-8.26

4. Application of strength criteria

The strength criterion is used for material mechanical behaviour evaluation (Žiliukas 2007):

$$\sigma_1 - \chi\sigma_2 \le \sigma_{u,t}, \chi = \frac{\sigma_{u,t}}{\sigma_{u,c}}, \quad (8)$$

$\sigma_{u,t}$ is the ultimate tension strength; and $\sigma_{u,c}$ is the limit compression strength.

Besides the strength criterion of Drucker and Prager may be applied (Žiliukas 2007):

$$\left(\sigma_{u,c} + \sigma_{u,t}\right)^2 \sigma_i \le \left[\sigma_{u,t}\sqrt{\sigma_{u,c}} + \sigma_{u,c}\sqrt{\sigma_{u,t}} - 3\left(\sqrt{\sigma_{u,c}} - \sqrt{\sigma_{u,t}}\right)\right]^2,$$

where

$$\left(\sigma_{u,c} + \sigma_{u,t}\right)^2 \sigma_i \le \left[\sigma_{u,t}\sqrt{\sigma_{u,c}} + \sigma_{u,c}\sqrt{\sigma_{u,t}} - 3\left(\sqrt{\sigma_{u,c}} - \sqrt{\sigma_{u,t}}\right)\right]^2,$$

$$\sigma_0 = (\sigma_1 + \sigma_2 + \sigma_3)/3.$$

These criteria should be applied for a mixed load, i.e. in case of stretching and compression, however. In case of contact load, spatial compression is encountered. Then it is more meaningful to apply the strength criterion of A. Žiliukas, which is written as follows (Žiliukas 2007):

$$m_1\sigma_i + m_2\sigma_0 \le \sigma_{u,\mu_\sigma}, \quad (10)$$

where m_1 and m_2 are material constants when μ=const and

$$\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}. \quad (11)$$

When the characteristics of compression tests are applied, the limit stress $\sigma_{u,\ m\sigma}$ is calculated as follows:

$$\sigma_{u,\mu_\sigma} = \frac{\sigma_{u,c_1} - \sigma_{u,c_{12}}}{2}\left(\mu_\sigma + 1\right) + \sigma_{u,c_{12}}, \qquad (12)$$

where $\sigma_{u,c1}$ is the strength limit in case of uniaxial compression; $\sigma_{u,c12}$ is the strength limit in case of biaxial compression.

As the tangential stresses $\sigma_{12} = \sigma_{23} = \sigma_{31} = 0$, thus σ_{12}, σ_{23} and σ_{31} are the principal and $\sigma_{11} = \sigma_1$; $\sigma_{22} = \sigma_2$; and $\sigma_{33} = \sigma_3$.

Thus in order to apply the criterion (10) it is necessary to find the material constants $\sigma_{u,c1}$ and $\sigma_{u,c12}$.

When the tests were done for uniaxial compression and biaxial compression, the strength limits for 7X2 steel were the following:

σ_u, c_1 = 2100 MPa.

σ_u, c_{12} = 3675 MPa.

Where

$$\sigma_{u,\mu_\sigma} = \frac{-2100 + 3675}{2}(\mu_\sigma + 1) - 3675 =$$
$$787{,}5\left(\mu_\sigma + 1\right) - 3675 MPa.$$

If $\sigma_1 = \sigma_2$

$$\mu_\sigma = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = 1.$$

Thus $\sigma_{u,\, m\sigma}$ = 2100 MPa.

The intensity of stress σ_i that is calculated from the formula is equal to

$$\sigma_i = \sigma_1 - \sigma_3. \qquad (13)$$

While the average tension is

$$\sigma_0 = \frac{2\sigma_1 + \sigma_3}{3}. \qquad (14)$$

Then criterion (10) is written in the following way:

$$m_1\left(\sigma_1 - \sigma_3\right) + m_2\frac{2\sigma_1 + \sigma_3}{3} \le \sigma_{u,c_1}.$$

where

$$\left(3m_1 + 2m_2\right)\sigma_1 + \left(m_2 - 3m_1\right)\sigma_3 \le \sigma_{u,c_1}. \qquad (15)$$

When $\sigma_1 = \sigma_u$, c_1 and $\sigma_3 = 0$, we receive

$$3m_1 + 2m_2 = 1. \qquad (16)$$

When $\sigma_3 = \sigma_{u,\,c2}$, and $\sigma_1 = 0$

$$\left(m_2 - 3m_1\right)\sigma_{u,c_2} = \sigma_{u,c_1}. \qquad (17)$$

From (16) and (17) we receive the following:

$$m_2 = \frac{1 + \dfrac{\sigma_{u,c_1}}{\sigma_{u,c_2}}}{3} = m_1 = \frac{\sigma_{u,c_1}}{2\sigma_{u,c_2}}.$$

Thus after we have done the compression tests in two perpendicular directions, we receive the strength criterion that helps us evaluate the strength of contact.

In the cases analyzed earlier when σ_3 = -1400 MPa, σ_1 = -1120 MPa and σ_u, $c_1 = \sigma_u$, c_2 after we have checked criterion (15) we receive:

$m_1 = 1$, $m_2 = 0.5$ and $4\sigma_1 + 0.17\,\sigma_3 \le \sigma_u$, c_1

or 4718 MPa> 2100 MPa

When σ_3 = -700 MPa, σ_1 = -560 MPa, 2359 MPa> 2100 MPa

When σ_3 = -280 MPa, σ_1 = -224 MPa, 944 MPa<210 MPa

When σ_3 = -350 MPa, σ_1 = -280 MPa, 1995 MPa< 2100 MPa

In such a way the acceptable pressure in the contact area shall be calculated as follows:

$$p_0 = 2.8 \cdot 10^5 \cdot 1.25 \cdot 10^{-3} = 3500 MPa$$

The value received is the limit in the radial bearing in practice. This proves the acceptability of the methodology.

5. Conclusions

When problems relating to the strength of contact pairs in aviation structures are being solved, it is important to evaluate triaxial load. Usually such evaluation is done by introducing correction coefficients.

To evaluate the strength of contact pairs in this article, the strength criterion suitable for complex stress is used.

According to the results of calculations, the suggested criterion is acceptable for the evaluation of strength of contact pairs.

References

Filonenko, S.; Stadnychenko, V.; Stakhova, A. 2008. The acoustic emission signals modeling at friction of materials surface layers, *Aviation* 12(3): 87–94.

Filonenko, S.; Stadnychenko, V.; Stakhova, A. 2009. Definition of contract stress in friction units on the basis of the acoustic emission method, *Aviation* 13(3): 72–77.

Johnson, K. L. 1985. *Contact Mechanics*. London: Cambridge Univ. Press. 324 p.

Morozov, E. M.; Zernin, M. V. 1999. *Contact Problem of Fracture Mechanics.* Moscow: Machinostrenia. 544 p. (in Russian).

Žiliukas, A. 2006. *Strength and Fracture Criteria*. Kaunas: Technologija. 208 p. (in Lithuanian).

Žiliukas, A. 2007. *Theorie of Elasticity and Plasticity*. Kaunas: Technologija. 275 p. (in Lithuanian).

AVIACINIŲ KONSTRUKCIJŲ SFERINIŲ KONTAKTINIŲ PORŲ STIPRUMO ĮVERTINIMAS

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S a n t r a u k a

Kontaktinių porų stiprumui įvertinti taikomas stiprumo kriterijus, kuris naudojamas sudėtingo įtempių būvio atvejais. Ši metodika leidžia skaičiavimuose atsisakyti empirinių koeficientų. Gauti skaičiavimų rezultatai rodo kriterijaus priimtinumą, palyginti su eksperimentiniais.

Reikšminiai žodžiai: aviacinė konstrukcija, kontaktinė pora, stiprumo kriterijus.